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# On Reciprocal Complementary Wiener Index of a Graph

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**ABSTRACT**

The eccentricity of a vertex $v$ of $G$ is the largest distance between $v$ and any other vertex in $G$. The reciprocal complementary Wiener (*RCW*) index of $G$ is defined as

$$RCW(G) = \sum_{1 \le i < j \le n} \frac{1}{1+D-d(v_i,v_j)},$$

where $D$ is the diameter of $G$ and $d(v_i,v_j)$ is the distance between the vertices $v_i$ and $v_j$. In this paper, we have obtained bounds for the *RCW* index in terms of eccentricities and given an algorithm to compute the *RCW* index.



## 1 INTRODUCTION

Graph theory has provided chemist with a variety of useful tools, such as Topological Index. Molecules and molecular compounds are often modeled by molecular graph. A molecular graph is a representation of the structural formula of a chemical compound in terms of graph theory, whose vertices correspond to the atoms of the compound and edges correspond to the chemical bonds.

Throughout this paper we consider only simple, connected graphs without loops and multiple edges [1]. Let $G$ be such graph with $n$ vertices, $m$ edges and vertex set $V(G) = \{v_1, v_2, \ldots, v_n\}$. The *degree* of $v_i \in V(G)$, denoted by $deg(v_i)$, is the number of vertices adjacent to $v_i$. The sum of the degrees of the vertices of $G$ is $2m$. The *distance* between the vertices

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$v_i$ and $v_j$ of $V(G)$, denoted by $d(v_i, v_j)$, is the length of the shortest path joining them. The *eccentricity* of a vertex $v \in V(G)$, denoted by $e(v)$, is the largest distance between $v$ and any other vertex of the graph $G$. The *radius* $r = r(G)$ of $G$ is the minimum eccentricity of the vertices and the *diameter* $D = D(G)$ of $G$ is the maximum eccentricity. A vertex $v$ is called *central vertex* of $G$, if $e(v) = r(G)$. A graph is called *self-centered* if every vertex is a central vertex. Thus in a self-centered graph $r(G) = D(G)$. A vertex $u$ is said to be an *eccentric vertex* of a vertex $v$ if $d(u, v) = e(v)$. An *eccentric path* $P(v)$ of a vertex $v$ is a path of length $e(v)$ joining $v$ and its eccentric vertex. There may exist more than one eccentric path for a given vertex.

A topological index is a graph invariant applicable in chemistry. The Wiener index is the first topological index introduced by Harold Wiener in 1947 [11]. There are many topological indices which are frequently made their appearance in both chemical and mathematical literature.

Wiener index $W(G)$ of a graph $G$ is defined as [11],

$$W(G) = \sum_{1 \leq i < j \leq n} d(v_i, v_j). \tag{1}$$

The *reciprocal complementary Wiener* (*RCW*) index of a graph $G$ is defined as [3, 4]

$$RCW(G) = \sum_{1 \leq i < j \leq n} \frac{1}{1 + D - d(v_i, v_j)}, \tag{2}$$

where $D$ is the diameter of $G$.

The reciprocal complementary distance number of vertex $v_i$ of $G$, denoted by $RCDN(v_i \mid G)$ is defined as,

$$RCDN(v_i \mid G) = \sum_{j=1}^{n} \frac{1}{1 + D - d(v_i, v_j)}.$$

Therefore, $RCW(G) = \frac{1}{2} \sum_{i=1}^{n} RCDN(v_i \mid G)$.

The chemical applications of *RCW* index are reported in the literature [3−5, 10] and one can refer the mathematical properties of *RCW* index in [2, 6, 8, 12−14]. *RCW* index has been successfully applied in the structure property modeling of the molar heat capacity, standard Gibbs energy of formation and vaporization enthalpy of 134 alkanes $C_6 - C_{10}$ [3]. In [2] Cai and Zhou determined the trees with the smallest, the second smallest and the third smallest *RCW* indices, and the unicyclic and bicyclic graphs with the smallest and the second smallest *RCW* indices. In [13] Zhou et al. obtained some properties, especially various upper and lower bounds and Nordhaus-Gaddum-type results of *RCW* indices. Qi and Zhou [6] characterized the trees with fixed number of vertices and matching number with the smallest *RCW* index. Ramane et al. [7, 9] obtained bounds for the Wiener number and also for Harary index in terms of eccentricities. The present work contains bounds on the $RCW$ index in terms of eccentricities and moreover, we have given a simple algorithm to compute $RCW$ index for any simple graph.

## 2. MAIN RESULTS

**Theorem 1.** Let $G$ be a simple, connected graph with $n$ vertices, $m$ edges, diameter $D$ and $e_i = e(v_i)$, for $i = 1,2,...,n$. Then,

$$RCW(G) \geq \frac{1}{2}\left[\frac{n(n-1)-\sum_{i=1}^{n} e_i}{D-1} - \frac{2m-n}{D(D-1)} + \sum_{i=1}^{n}\sum_{j=1}^{e_i}\frac{1}{D-(j-1)}\right]. \tag{3}$$

Equality holds if and only if for every vertex $v_i$ of $G$, if $P(v_i)$ is one of the eccentric path of $v_i$, then for every $v_j \in V(G)$ which is not on $P(v_i)$, $d(v_i, v_j) \leq 2$.

**Proof.** Let $P(v_i)$ be one of the eccentric path of $v_i \in V(G)$. Let

$A_1(v_i) = \{v_j \mid v_j$ is on eccentric path $P(v_i)$ of $v_i\}$,

$A_2(v_i) = \{v_j \mid v_j$ is adjacent to $v_i$ and which is not on the eccentric path $P(v_i)$ of $v_i\}$,

$A_3(v_i) = \{v_j \mid v_j$ is not adjacent to $v_i$ and not on the eccentric path $P(v_i)$ of $v_i\}$.

It is clear that $A_1(v_i) \cup A_2(v_i) \cup A_3(v_i) = V(G)$ and $|A_1(v_i)| = e_i + 1$, $|A_2(v_i)| = deg(v_i) - 1$, $|A_3(v_i)| = n - e_i - deg(v_i)$. Now

$$\sum_{v_j \in A_1(v_i)} \frac{1}{1+D-d(v_i,v_j)} = \sum_{j=1}^{e_i}\frac{1}{D-(j-1)}, \quad \sum_{v_j \in A_2(v_i)} \frac{1}{1+D-d(v_i,v_j)} = \frac{deg(v_i)-1}{D},$$

$$\sum_{v_j \in A_3(v_i)} \frac{1}{1+D-d(v_i,v_j)} \geq \frac{n-e_i-deg(v_i)}{D-1}.$$

Therefore,

$$\begin{aligned} RCDN(v_i \mid G) &= \sum_{j=1}^{n}\frac{1}{1+D-d(v_i,v_j)} \\ &= \sum_{v_j \in A_1(v_i)} \frac{1}{1+D-d(v_i,v_j)} + \sum_{v_j \in A_2(v_i)} \frac{1}{1+D-d(v_i,v_j)} + \\ &\quad \sum_{v_j \in A_3(v_i)} \frac{1}{1+D-d(v_i,v_j)} \\ &\geq \frac{D(n-e_i-1)-deg(v_i)+1}{D(D-1)} + \sum_{j=1}^{e_i}\frac{1}{D-(j-1)}. \end{aligned}$$

Therefore,

$$RCW(G)=\frac{1}{2}\sum_{i=1}^{n}RCDN(v_i\mid G)$$
$$\geq\frac{1}{2}\sum_{i=1}^{n}\left[\frac{D(n-e_i-1)-deg(v_i)+1}{D(D-1)}+\sum_{j=1}^{e_i}\frac{1}{D-(j-1)}\right]$$
$$=\frac{1}{2}\left[\frac{n(n-1)-\sum_{i=1}^{n}e_i}{D-1}-\frac{2m-n}{D(D-1)}+\sum_{i=1}^{n}\sum_{j=1}^{e_i}\frac{1}{D-(j-1)}\right].$$

For equality, Let $d(v_i,v_j)=2$, where $v_j\in A_3(v_i)$. Therefore

$$\sum_{v_j\in A_1(v_i)}\frac{1}{1+D-d(v_i,v_j)}=\sum_{j=1}^{e_i}\frac{1}{D-(j-1)},\qquad \sum_{v_j\in A_2(v_i)}\frac{1}{1+D-d(v_i,v_j)}=\frac{\deg(v_i)-1}{D},$$
$$\sum_{v_j\in A_3(v_i)}\frac{1}{1+D-d(v_i,v_j)}=\frac{n-e_i-deg(v_i)}{D-1}.$$

Thus

$$RCDN(v_i\mid G)=\sum_{j=1}^{n}\frac{1}{1+D-d(v_i,v_j)}$$
$$=\sum_{v_j\in A_1(v_i)}\frac{1}{1+D-d(v_i,v_j)}+\sum_{v_j\in A_2(v_i)}\frac{1}{1+D-d(v_i,v_j)}+$$
$$\sum_{v_j\in A_3(v_i)}\frac{1}{1+D-d(v_i,v_j)}$$
$$=\frac{D(n-e_i-1)-deg(v_i)+1}{D(D-1)}+\sum_{j=1}^{e_i}\frac{1}{D-(j-1)}.$$

Hence

$$RCW(G)=\frac{1}{2}\sum_{i=1}^{n}RCDN(v_i\mid G)$$
$$=\frac{1}{2}\left[\frac{n(n-1)-\sum_{i=1}^{n}e_i}{D-1}-\frac{2m-n}{D(D-1)}+\sum_{i=1}^{n}\sum_{j=1}^{e_i}\frac{1}{D-(j-1)}\right].$$

Conversely, suppose $G$ is not such as explained in the equality part of this theorem. Then there exist at least one vertex $v_j\in A_3(v_i)$ such that $d(v_i, v_j)\geq 3$. Let $A_3(v_i)$ be partitioned into two sets $A_{31}(v_i)$ and $A_{32}(v_i)$, where

$A_{31}(v_i)=\{v_j \mid v_j$ is not adjacent to $v_i$, not on the eccentric path $P(v_i)$ of $v_i$ and $d(v_i, v_j)=2\}$,

$A_{32}(v_i) = \{v_j \mid v_j$ is not adjacent to $v_i$, not on the eccentric path $P(v_i)$ of $v_i$ and $d(v_i, v_j) \geq 3\}$. Let $|A_{32}(v_i)| = l \geq 1$. So, $|A_{31}(v_i)| = n - e_i - deg(v_i) - l$. Therefore

$$\sum_{v_j \in A_1(v_i)} \frac{1}{1+D-d(v_i,v_j)} = \sum_{j=1}^{e_i} \frac{1}{D-(j-1)}, \quad \sum_{v_j \in A_2(v_i)} \frac{1}{1+D-d(v_i,v_j)} = \frac{\deg(v_i)-1}{D},$$

$$\sum_{v_j \in A_{31}(v_i)} \frac{1}{1+D-d(v_i,v_j)} = \frac{n-e_i-deg(v_i)-l}{D-1}, \quad \sum_{v_j \in A_{32}(v_i)} \frac{1}{1+D-d(v_i,v_j)} \geq \frac{l}{D-2}.$$

Therefore,

$$\begin{aligned} RCDN(v_i \mid G) &= \sum_{j=1}^{n} \frac{1}{1+D-d(v_i,v_j)} \\ &= \sum_{v_j \in A_1(v_i)} \frac{1}{1+D-d(v_i,v_j)} + \sum_{v_j \in A_2(v_i)} \frac{1}{1+D-d(v_i,v_j)} + \\ &\quad \sum_{v_j \in A_{31}(v_i)} \frac{1}{1+D-d(v_i,v_j)} + \sum_{v_j \in A_{32}(v_i)} \frac{1}{1+D-d(v_i,v_j)} \\ &= \frac{D(n-e_i-1)-deg(v_i)+1}{D(D-1)} + \sum_{j=1}^{e_i} \frac{1}{D-(j-1)} \\ &\quad + \frac{l}{(D-2)(D-1)} \end{aligned}$$

and so

$$\begin{aligned} RCW(G) &= \frac{1}{2}\sum_{i=1}^{n} RCDN(v_i \mid G) \\ &\geq \frac{1}{2}\sum_{i=1}^{n}\left[\frac{D(n-e_i-1)-deg(v_i)+1}{D(D-1)} + \sum_{j=1}^{e_i}\frac{1}{D-(j-1)} + \frac{l}{(D-2)(D-1)}\right] \\ &= \frac{1}{2}\left[\frac{n(n-1)-\sum_{i=1}^{n} e_i}{D-1} - \frac{2m-n}{D(D-1)} + \sum_{i=1}^{n}\sum_{j=1}^{e_i}\frac{1}{D-(j-1)} + \frac{nl}{(D-2)(D-1)}\right]. \end{aligned}$$

This is a contradiction to the equality as $l \geq 1$. This completes the proof. □

**Corollary 2.** Let $G$ be a self-centered graph with $n$ vertices, $m$ edges and radius $r = r(G)$. Then

$$RCW(G) \geq \frac{1}{2}\left[\frac{nr(n-1-r)-2m+n}{r(r-1)} + n\sum_{j=1}^{r}\frac{1}{r-(j-1)}\right]. \tag{4}$$

Equality holds if and only if for every vertex $v_i$ of a self-centered graph $G$, if $P(v_i)$ is one of the eccentric path of $v_i$ then for every $v_j \in V(G)$ which is not on the eccentric path $P(v_i)$, $d(v_i, v_j) \leq 2$.

**Proof.** Since $G$ is a self-centered graph, the radius $r = e_i = e(v_i) = D$ for $i = 1, 2, \ldots, n$. Therefore by Eq. (3)

$$RCW(G) \geq \frac{1}{2}\left[\frac{n(n-1-r)}{r-1} - \frac{2m-n}{r(r-1)} + \sum_{i=1}^{n}\sum_{j=1}^{e_i}\frac{1}{r-(j-1)}\right]$$
$$= \frac{1}{2}\left[\frac{nr(n-1-r)-2m+n}{r(r-1)} + n\sum_{j=1}^{r}\frac{1}{r-(j-1)}\right].$$

Equality part can be proved in analogous to the proof of equality part of Theorem 1. □

**Theorem 3.** Let $G$ be a connected graph with $n$ vertices and $e_i = e(v_i)$, $i = 1, 2, \ldots, n$. Then

$$RCW(G) \geq \frac{1}{2}\left[\frac{n(n-1) - \sum_{i=1}^{n} e_i}{D} + \sum_{i=1}^{n}\sum_{j=1}^{e_i}\frac{1}{D-(j-1)}\right]. \tag{5}$$

Equality holds if and only if for every vertex $v_i$ of $G$, if $P(v_i)$ is one of the eccentric path of $v_i$, then for every $v_j \in V(G)$ which is not on $P(v_i)$, $d(v_i, v_j) = 1$.

**Proof.** Let $P(v_i)$ be one of the eccentric path of $v_i \in V(G)$, $B_1(v_i) = \{v_j \mid v_j$ is on eccentric path $P(v_i)$ of $v_i\}$ and $B_2(v_i) = \{v_j \mid v_j$ is not on the eccentric path $P(v_i)$ of $v_i\}$. It is easy to check that $B_1(v_i) \cup B_2(v_i) = V(G)$, $|B_1(v_i)| = e_i + 1$ and $|B_2(v_i)| = n - e_i - 1$. Now

$$\sum_{v_j \in B_1(v_i)} \frac{1}{1+D-d(v_i,v_j)} = \sum_{j=1}^{e_i}\frac{1}{D-(j-1)}, \quad \sum_{v_j \in B_2(v_i)} \frac{1}{1+D-d(v_i,v_j)} \geq \frac{n-e_i-1}{D}.$$

Therefore

$$RCDN(v_i \mid G) = \sum_{j=1}^{n}\frac{1}{1+D-d(v_i,v_j)}$$
$$= \sum_{v_j \in B_1(v_i)} \frac{1}{1+D-d(v_i,v_j)} + \sum_{v_j \in B_2(v_i)} \frac{1}{1+D-d(v_i,v_j)}$$
$$\geq \sum_{j=1}^{e_i}\frac{1}{D-(j-1)} + \frac{n-e_i-1}{D}.$$

Therefore

$$RCW(G) = \frac{1}{2}\sum_{i=1}^{n} RCDN(v_i \mid G)$$

$$\geq \frac{1}{2}\sum_{i=1}^{n}\left[\sum_{j=1}^{e_i}\frac{1}{D-(j-1)} + \frac{n-e_i-1}{D}\right]$$

$$= \frac{1}{2}\left[\frac{n(n-1)-\sum_{i=1}^{n} e_i}{D} + \sum_{i=1}^{n}\sum_{j=1}^{e_i}\frac{1}{D-(j-1)}\right].$$

For equality, let $d(v_i,v_j) = 1$, where $v_j \in B_2(v_i)$. Hence

$$\sum_{v_j \in B_1(v_i)} \frac{1}{1+D-d(v_i,v_j)} = \sum_{j=1}^{e_i}\frac{1}{D-(j-1)} \text{ and } \sum_{v_j \in B_2(v_i)} \frac{1}{1+D-d(v_i,v_j)} = \frac{n-e_i-1}{D}.$$

Therefore

$$RCDN(v_i \mid G) = \sum_{j=1}^{n}\frac{1}{1+D-d(v_i,v_j)}$$

$$= \sum_{v_j \in B_1(v_i)} \frac{1}{1+D-d(v_i,v_j)} + \sum_{v_j \in B_2(v_i)} \frac{1}{1+D-d(v_i,v_j)}$$

$$= \sum_{j=1}^{e_i}\frac{1}{D-(j-1)} + \frac{n-e_i-1}{D}.$$

Therefore

$$RCW(G) = \frac{1}{2}\sum_{i=1}^{n} RCDN(v_i \mid G)$$

$$= \frac{1}{2}\left[\frac{n(n-1)-\sum_{i=1}^{n} e_i}{D} + \sum_{i=1}^{n}\sum_{j=1}^{e_i}\frac{1}{D-(j-1)}\right].$$

Conversely, suppose $G$ is not a such graph as explained in the equality part of this theorem. Then there exist at least one vertex $v_j \in B_2(v_i)$ such that $d(v_i, v_j) \geq 2$. Let $B_2(v_i)$ be partitioned into two sets $B_{21}(v_i)$ and $B_{22}(v_i)$, where $B_{21}(v_i) = \{v_j \mid v_j$ is not on the eccentric path $P(v_i)$ of $v_i$ and $d(v_i, v_j) = 1\}$, $B_{22}(v_i) = \{v_j \mid v_j$ is not on the eccentric path $P(v_i)$ of $v_i$ and $d(v_i, v_j) \geq 2\}$. Let $|B_{22}(v_i)| = l \geq 1$ and $|B_{21}(v_i)| = n - e_i - 1 - l$. Therefore

$$\sum_{v_j \in B_1(v_i)} \frac{1}{1+D-d(v_i,v_j)} = \sum_{j=1}^{e_i} \frac{1}{D-(j-1)}, \quad \sum_{v_j \in B_{21}(v_i)} \frac{1}{1+D-d(v_i,v_j)} = \frac{n-e_i-1-l}{D},$$

$$\sum_{v_j \in B_{22}(v_i)} \frac{1}{1+D-d(v_i,v_j)} \geq \frac{l}{D-1}.$$

Therefore

$$RCDN(v_i \mid G) = \sum_{j=1}^{n} \frac{1}{1+D-d(v_i,v_j)}$$

$$= \sum_{v_j \in B_1(v_i)} \frac{1}{1+D-d(v_i,v_j)} + \sum_{v_j \in B_{21}(v_i)} \frac{1}{1+D-d(v_i,v_j)} + \sum_{v_j \in B_{22}(v_i)} \frac{1}{1+D-d(v_i,v_j)}$$

$$= \sum_{j=1}^{e_i} \frac{1}{D-(j-1)} + \frac{n-e_i-1-l}{D} + \frac{l}{D-1}.$$

Therefore

$$RCW(G) = \frac{1}{2}\sum_{i=1}^{n} RCDN(v_i \mid G)$$

$$\geq \frac{1}{2}\sum_{i=1}^{n}\left[\sum_{j=1}^{e_i} \frac{1}{D-(j-1)} + \frac{n-e_i-1-l}{D} + \frac{l}{D-1}\right]$$

$$= \frac{1}{2}\left[\sum_{i=1}^{n}\sum_{j=1}^{e_i} \frac{1}{D-(j-1)} + \frac{n(n-1)-\sum_{i=1}^{n} e_i}{D} + \frac{nl}{D(D-1)}\right].$$

As $l \geq 1$, it contradicts to the equality. This completes the proof. □

If $G$ is a self-centered graph then $e_i = e(v_i) = r(G)$ for all $i = 1, 2, \ldots, n$. Substituting this in Eq. (5) we get the following corollary.

**Corollary 4.** Let $G$ be a self-centered graph with $n$ vertices and radius $r = r(G)$. Then

$$RCW(G) \geq \frac{1}{2}\left[\frac{n(n-1-r)}{r} + n\sum_{j=1}^{r} \frac{1}{r-(j-1)}\right]. \tag{6}$$

Equality holds if and only if for every vertex $v_i$ of a self-centered graph $G$, if $P(v_i)$ is one of the eccentric path of $v_i$ then for every $v_j \in V(G)$ which is not on the eccentric path $P(v_i)$, then $d(v_i, v_j) = 1$.

**Theorem 5.** Let $G$ be a connected graph with $n$ vertices, $m$ edges and diameter $D$. Let $e_i$ = e($v_i$), $i$= 1, 2, … ,$n$. Then

$$RCW(G) \le \frac{1}{2}\left[n^2 - \sum_{i=1}^{n} e_i - 2m + \frac{2m-n}{D} + \sum_{i=1}^{n}\sum_{j=1}^{e_i}\frac{1}{D-(j-1)}\right]. \tag{7}$$

Equality holds if and only if $D \le 2$.

**Proof.** Let $P(v_i)$ be one of the eccentric path of $v_i \in V(G)$. Let
$A_1(v_i) = \{v_j \mid v_j$ is on the eccentric path $P(v_i)$ of $v_i\}$,
$A_2(v_i) = \{v_j \mid v_j$ is adjacent to $v_i$ and which is not on the eccentric path $P(v_i)$ of $v_i\}$,
$A_3(v_i) = \{v_j \mid v_j$ is not adjacent to $v_i$ and not on the eccentric path $P(v_i)$ of $v_i\}$.
It is easy to check that $A_1(v_i) \cup A_2(v_i) \cup A_3(v_i) = V(G)$ and $|A_1(v_i)| = e_i + 1$, $|A_2(v_i)| = deg(v_i) - 1$ and $|A_3(v_i)| = n - e_i - deg(v_i)$. Now

$$\sum_{v_j \in A_1(v_i)} \frac{1}{1+D-d(v_i,v_j)} = \sum_{j=1}^{e_i}\frac{1}{D-(j-1)}, \qquad \sum_{v_j \in A_2(v_i)} \frac{1}{1+D-d(v_i,v_j)} = \frac{deg(v_i)-1}{D},$$

$$\sum_{v_j \in A_3(v_i)} \frac{1}{1+D-d(v_i,v_j)} \le n - e_i - deg(v_i).$$

Therefore

$$\begin{aligned}
RCDN(v_i \mid G) &= \sum_{j=1}^{n}\frac{1}{1+D-d(v_i,v_j)} \\
&= \sum_{v_j \in A_1(v_i)} \frac{1}{1+D-d(v_i,v_j)} + \sum_{v_j \in A_2(v_i)} \frac{1}{1+D-d(v_i,v_j)} + \\
&\quad \sum_{v_j \in A_3(v_i)} \frac{1}{1+D-d(v_i,v_j)} \\
&\le \sum_{j=1}^{e_i}\frac{1}{D-(j-1)} + \frac{deg(v_i)-1}{D} + (n-e_i-deg(v_i)) \\
&= \frac{D(n-e_i)+(1-D)deg(v_i)-1}{D} + \sum_{j=1}^{e_i}\frac{1}{D-(j-1)}.
\end{aligned}$$

Thus

$$\begin{aligned}
RCW(G) &= \frac{1}{2}\sum_{i=1}^{n} RCDN(v_i \mid G) \\
&\le \frac{1}{2}\sum_{i=1}^{n}\left[\frac{D(n-e_i)+(1-D)deg(v_i)-1}{D} + \sum_{j=1}^{e_i}\frac{1}{D-(j-1)}\right]
\end{aligned}$$

$$= \frac{1}{2}\left[ n^2 - \sum_{i=1}^{n} e_i - 2m + \frac{2m-n}{D} + \sum_{i=1}^{n}\sum_{j=1}^{e_i} \frac{1}{D-(j-1)} \right].$$

For equality, let $D \le 2$. We consider here two cases.

Case 1: If $D = 1$, then $G = K_n$, a complete graph on $n$ vertices. Therefore, $A_3(v_i)$ is an empty set. Hence

$$RCW(G) = \frac{1}{2}\left[ n^2 - \sum_{i=1}^{n} e_i - n + \sum_{i=1}^{n} 1 \right] = \frac{n(n-1)}{2} .$$

Case 2: If $D = 2$, then for $v_j \in A_3(v_i)$, $d(v_i, v_j) = 2$. Therefore,

$$\sum_{v_j \in A_3(v_i)} \frac{1}{1+D-d(v_i,v_j)} = n - e_i - deg(v_i) .$$

Hence

$$RCW(G) = \frac{1}{2}\left[ n\left( n - \frac{1}{2} \right) - \sum_{i=1}^{n} e_i - m + \sum_{i=1}^{n}\sum_{j=1}^{e_i} \frac{1}{3-j} \right].$$

Conversely,

$$RCDN(v_i \mid G) = \sum_{j=1}^{n} \frac{1}{1+D-d(v_i,v_j)}$$

$$= \sum_{v_j \in A_1(v_i)} \frac{1}{1+D-d(v_i,v_j)} + \sum_{v_j \in A_2(v_i)} \frac{1}{1+D-d(v_i,v_j)} + \sum_{v_j \in A_3(v_i)} \frac{1}{1+D-d(v_i,v_j)} \qquad (8)$$

The first summation of Eq. (8) contains the distance between $v_i$ and the vertices on its eccentric path $P(v_i)$. Second summation of Eq. (8) contains the distance between $v_i$ and its neighbor which are not on the eccentric path $P(v_i)$. The third summation of Eq. (8) contains the distance between $v_i$ and a vertex which is neither adjacent to $v_i$ nor on the eccentric path $P(v_i)$. Hence the equality in Eq. (8) holds if and only if $D \le 2$. It is true for all $v_i \in V(G)$, which completes the proof. □

**Corollary 6.** Let $G$ be a self-centered graph with $n$ vertices and radius $r = r(G)$. Then

$$RCW(G) \le \frac{1}{2}\left[ n^2 - nr - 2m + \frac{2m-n}{r} + n\sum_{j=1}^{e_i} \frac{1}{r-(j-1)} \right].$$

Equality holds if and only if $D \le 2$.

**Proof.** Follows by substituting $e_i = e(v_i) = r$, for $i = 1, 2, \ldots, n$ in Theorem 5. □

**Algorithm:** To compute RCW index

Distance matrix of a graph $G$ is a matrix $Dt(G) = [d_{ij}]$ of order $n$, where $d_{ij} = d(v_i, v_j)$.
**Input:** Distance matrix of a given graph.
Step 1: Declared d[i ] [ j ], *rc* [ i ] [ j ], $D = 0$, $RCW = 0$, Sum = 0.
Step2: Read the distance matrix of order $n$.
Step 3: For $i \to 1$ to $n$
    For $j \to 1$ to $n$
    if (*d*[i ] [ j ] >*D*)
    *D*→ *d*[ i ] [ j ].
Step 4: For $i \to 1$ to $n$
    For $j \to 1$ to $n$
    Set *rc*[i ] [ j ] = 0 if $i = j$ and *rc*[ i ][ j ] = $1/(1+D - d_{ij})$, otherwise.
    Sum= Sum + *rc*[i ] [ j ].
Step 5: Compute *RCW* = Sum divided by 2.
Step 6: Display *RCW*.
**Output:** *RCW* index of given graph.

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