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Computing PI and Hyper–Wiener Indices of Corona Product of some Graphs

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ABSTRACT

Let G and H be two graphs. The corona product G o H is obtained by taking one copy of G and |V(G)| copies of H; and by joining each vertex of the i-th copy of H to the i-th vertex of G, i = 1, 2, …, |V(G)|. In this paper, we compute PI and hyper–Wiener indices of the corona product of graphs.

Keywords: Topological indices; PI index; hyper−Wiener index; Wiener index

1. INTRODUCTION

Let *G be* a connected graph with vertex and edge sets V(G) and E(G), respectively. As usual, the distance between the vertices u and v *o*f G is denoted by d(u,v) and it is defined as the number of edges in a minimal path connecting the vertices u and v. A topological index is a real number related to a graph. It must be a structural invariant, i.e., it preserves by every graph automorphisms. There are several topological indices have been defined and many of them have found applications as means to model chemical, pharmaceutical and other properties of molecules.

The Wiener index, W, is the first topological index to be used in chemistry. Usage of topological indices in chemistry began in 1947 when Harold Wiener developed the Wiener index and used it to determine physical properties of types of alkanes known as paraffin [17]. In a graph theoretical language, the Wiener index is equal to the count of all shortest distances in a graph. We encourage the reader to consult the special issues of MATCH Communication in Mathematics and in Computer Chemistry [2], Discrete Applied Mathematics [5] and [3,4], for more information on results on the Wiener index, the chemical meaning of the index and its history.

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The hyper-Wiener index of acyclic graphs was introduced by Milan Randić in 1993. Then Klein et al. [1], generalized Randić's definition for all connected graphs, as a generalization of the Wiener index. It is defined as

$$WW(G)=\frac{1}{2}W(G)+\frac{1}{2}\sum_{\{u,v\}\subseteq V(G)} d^2(u,v).$$

Let G be a graph and e = uv an edge of G. $n_{eu}(e\,|\,G)$ denotes the number of edges lying closer to the vertex u than the vertex v, and $n_{ev}(e\,|\,G)$ is the number of edges lying closer to the vertex v than the vertex u. The Padmakar–Ivan (PI) index of G is defined as $PI(G)=\Sigma_{e\in E(G)}[n_{eu}(e|G)+n_{ev}(e|G)]$, see [6–10,11–13] for details. In this definition, edges equidistant from both ends of the edge e = uv are not counted. We call this index the edge PI index and denote it by $PI_e(G)$. We also define the vertex PI index of G, $PI_v(G)$, as the sum of $[m_{eu}(e|G)+m_{ev}(e|G)]$ over all edges of G, where $m_{eu}(e\,|\,G)$ is the number of vertices lying closer to the vertex u than the vertex v and $m_{ev}(e\,|\,G)$ is the number of vertices lying closer to the vertex v than the vertex u. The Wiener index of Corona of graphs was studied in [14]. In [15], Dragan Stevanović computed the Hosoya polynomial of this graph operation. Here we continue this progress to compute the PI and hyper–Wiener indices of this graph. The main result of this paper is as follows:

2. MAIN RESULTS

In this section, some exact formulas for the PI and hyper-Wiener indices of the corona product of some graphs are presented. We begin with the following crucial lemma related to distance properties of corona of graphs.

Lemma 1. Let G and H be graphs and $N_G(e)=|E(G)|-(n_{eu}(e|G)+n_{ev}(e|G))$ where e = uv. Thus, we have

(a) $\forall e=uv\in E(GoH)$ such that $u\in V(G), v\in V(H_i)$, (i=1,…, $|V(G)|$), we have $N_{GoH}(e)=k(k-1)$.

(b) $\forall e\in H_i$, (i=1,…, $|V(G)|$), we have

$$N_{GoH}(e)=\frac{mnk}{2}-2k(k-1)+(n-2)+|E(G)|+(m-1)n.$$

(c) $\forall e=uv\in E(GoH)$ such that $u,v\in V(G)$, we have

$$N_{GoH}(e)=\left(m-(m_{eu}(e|G)+m_{ev}(e|G))\right)\left(n+\frac{nk}{2}\right)+N_G(e).$$

(d) $\forall xu, yv \in E(GoH)$ such that $x \in V(H_i), y \in V(H_j), (i \neq j)$ and $u, v \in V(G)$ we have $d_{GoH}(x,y) = d_G(u,v) + 2$, $d_{GoH}(x,v) = d_G(u,v) + 1$ and $d_{GoH}(y,u) = d_G(u,v) + 1$.

Proof. The proof is straightforward and so omitted.

A regular graph is a graph where each vertex has the same number of neighbors. A regular graph with vertices of degree k is called a k-regular graph or regular graph of degree k. A cycle is a circuit in which no vertex except the first (which is also the last) appears more than once. A square-free graph is a graph containing no graph cycles of length four. A triangle-free graph is a graph containing no graph cycles of length three.

Theorem 1. Let G be a graph with m vertex and H be a k–regular, square–free and triangle–free graph which $|V(H)|$ = n. Thus, we have:

$$PI_e(GoH) = PI_e(G) + \frac{n}{2}(k+2)PI_v(G) + \frac{m^2n^2}{2}(k+2) + mnk(k^2 - 2k + 2) + mn(|E(G)| - 1).$$

Proof. By Lemma 1, if A is the number of parallel edges to an edge of copies of H, then we have

$$A = \frac{nk}{2} - 2k(k-1) + (n-2) + |E(G)| + n(m-1) + \frac{nk}{2}(m-1).$$

On the other hand, by Lemma 1, if B is the number of parallel edges to an edge of G, then we have $B = |E(G)|^2 + mn\left(\frac{k}{2}+1\right)|E(G)| - PI_e(G) - n\left(\frac{k}{2}+1\right)PI_v(G)$. Let

$$C = \left| \bigcup_{\substack{uv \in E(GoH) \\ v \in V(H), u \in V(G)}} \{f \in E(GoH) \mid f \parallel uv\} \right|,$$

then C = mn(k(k – 1) + 1). By summation of A, B and C, the result can be proved.

Theorem 2. Let G and H be graphs. Thus, we have

$$WW(GoH) = (|V(H)| + 1)^2\, WW(G) + 2|V(H)|(|V(G)| + |V(H)|)W(G)$$

$$-\frac{|V(H)||V(G)|}{2}\left(3|V(H)||V(G)|+|V(G)|^2+|V(G)|-3\right)-2|V(G)||E(H)|.$$

Proof. Let H_i be the i-th copy of H, i = 1, 2, …, $|V(G)|$. Then

$$\forall a,b\in V(H_i), d(a,b)=\begin{cases}1 & ab\in E(H)\\ 2 & ab\notin E(H)\end{cases}.$$

Now the summation of distances between the vertices is equal to

$$-4|V(G)||E(H)|+3|V(G)||V(H)|^2-3|V(G)||V(H)| \qquad (1)$$

By Lemma 1, for the summation of distances between the vertices of H_i's

$$5|V(H)|^2 W(G)+3|V(G)|^2|V(H)|^2-3|V(G)||V(H)|^2+|V(H)|^2\sum_{\{u,v\}\subseteq V(G)} d^2(u,v). \qquad (2)$$

Also, for the summation of distances between the inner vertices of G,

$$W(G)+\sum_{\{u,v\}\subseteq V(G)} d^2(u,v) \qquad (3)$$

and by Lemma 1, for the summation of distances between the vertices of G from H_i,

$$2|V(H)|\sum_{\{u,v\}\subseteq V(G)} d^2(u,v)+2|V(H)|W(G)+4|V(G)||V(H)|W(G)$$

$$+|V(G)|^3|V(H)|+|V(G)|^2|V(H)| \qquad (4)$$

By Equations 1, 2, 3 and 4, the result is proved.

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