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Comment on "Lower and Upper Bounds between Energy, Laplacian Energy, and Sombor Index of Some Graphs" [Iranian J. Math. Chem. 16 (2025) 33-38]

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Abstract

This note corrects several errors in the published paper (Barzegar, 2025). We identify the main flaws and present corrected versions of the affected results.

1 Introduction

Let $G = (V, E)$ be a simple undirected graph with n vertices and m edges. Let Δ be its maximum degree. The adjacency eigenvalues λ_i and Laplacian eigenvalues μ_i define the energy and Laplacian energy as:

$$\epsilon(G) = \sum_{i=1}^{n} |\lambda_i|, \qquad L\epsilon(G) = \sum_{i=1}^{n} \left|\mu_i - \frac{2m}{n}\right|.$$

The Sombor index is

$$SO(G) = \sum_{uv \in E} \sqrt{d_u^2 + d_v^2}.$$

We point out several fundamental errors in [1] and provide corrections.

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2 Main errors identified

2.1 Systematic omission of $m > 0$

In Theorems 2.3, 2.10, 2.11, 2.12, 2.14, Lemma 2.9, and Corollary 2.4 of [1], the author uses fractions with denominators involving m or Δ. For edgeless graphs ($m = 0$, $\Delta = 0$), these expressions are undefined. Hence, all these statements are valid only for graphs with $m > 0$ (equivalently, $\Delta > 0$). The corrected statements must explicitly assume $m > 0$.

2.2 Theorem 2.3 (equality condition)

The original claim that equality in

$$\epsilon(G) \leq \frac{SO(G)}{2(m/n)^{3/2}},$$

holds for $G = \overline{K}_n$ is invalid. The correct equality case is only $G \cong \bigcup_{i=1}^{m} K_2$ when $m > 0$.

2.3 Lemma 2.9 (incomplete equality characterisation)

The inequality

$$\epsilon(G) \geq \frac{2m}{\Delta},$$

is valid for $\Delta > 0$. However, the equality condition given in [1] is incomplete. Equality holds if and only if every connected component of G is either an isolated vertex or a complete bipartite graph $K_{\Delta,\Delta}$. Equivalently,

$$G \cong \bigcup_{i=1}^{k} K_{\Delta,\Delta} \ \cup \ \overline{K}_t,$$

where $k \geq 0$, $t \geq 0$, and $\Delta > 0$. For completeness, we provide a proof of this lemma below.

Lemma 2.1 (Corrected Lemma 2.9 of [1]). *Let G be a simple graph with $\Delta > 0$. Then*

$$\epsilon(G) \geq \frac{2m}{\Delta},$$

where, equality is achieved if and only if G is a disjoint union of several complete bipartite graphs $K_{\Delta,\Delta}$ and $\overline{K_t}$, i.e., $\cup_{i=1}^{k} K_{\Delta,\Delta} \cup \overline{K_t}$.

Proof. Let $\lambda_1, \lambda_2, \ldots, \lambda_n$ be the eigenvalues of the adjacency matrix $A(G)$. By Gershgorin's theorem or the well-known fact that the spectral radius is at most Δ, we have

$$|\lambda_i| \leq \Delta \quad \text{for every } i = 1, 2, \ldots, n. \tag{1}$$

Multiplying by $|\lambda_i| \geq 0$, we obtain

$$\Delta|\lambda_i| \geq |\lambda_i|^2 = \lambda_i^2 \qquad (i = 1, 2, \ldots, n). \tag{2}$$

Summing over all i, we get

$$\Delta \sum_{i=1}^{n} |\lambda_i| \geq \sum_{i=1}^{n} \lambda_i^2. \tag{3}$$

By the definition of energy, which gives the desired inequality $\epsilon(G) \geq \frac{2m}{\Delta}$.

Equality holds in the sum if and only if equality holds in every individual inequality $\Delta|\lambda_i| \geq \lambda_i^2$. This occurs if and only if for each i, either $\lambda_i = 0$ or $|\lambda_i| = \Delta$. It is well-known that the only connected graphs for which all non-zero eigenvalues have absolute value Δ are complete bipartite graphs $K_{\Delta,\Delta}$ (this follows from the characterization of graphs with spectral radius Δ and exactly two distinct eigenvalues). For disconnected graphs, equality holds if and only if each non isolated component is such a graph. Thus $G = \cup_{i=1}^{k} K_{\Delta,\Delta} \cup \overline{K_t}$. ■

2.4 Corollary 2.13(i)

The claim that for any tree G, $\frac{\sqrt{2}m}{n} \geq 1$ is false. This inequality fails not only for K_2 (where $\frac{\sqrt{2}}{2} < 1$), but also for P_3 (where $\frac{2\sqrt{2}}{3} < 1$). Consequently, the inequality $L\epsilon(G) \leq SO(G)$ does not hold for all trees; it fails for K_2 and P_3. The correct statement must either restrict to trees satisfying $m \geq \frac{n}{\sqrt{2}}$, or explicitly exclude all trees with $m < \frac{n}{\sqrt{2}}$.

2.5 Theorem 2.14

Theorem 2.14 in [1] is false as stated. Even under the additional condition $n \geq 2m$, the inequality fails. For $G = K_2$, we have $n = 2m$, but

$$L\epsilon(K_2) = 2 < 2\sqrt{2} = \frac{SO(K_2)}{m/\sqrt{2n}},$$

which directly contradicts the claimed inequality. Therefore, the theorem is fundamentally invalid, and no corrected version is provided.

3 Conclusion

This note systematically corrects fundamental flaws in [1], emphasizing the importance of the $m > 0$ condition and providing rigorous mathematical corrections.

Conflicts of Interest. The authors declare that they have no conflicts of interest regarding the publication of this article.

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