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Research Paper

Chemical Hyperstructures for Neptunium, Rubidium, and Plutonium

Fakhry Asad Agusfrianto[1], Mariam Hariri[2] and Yudi Mahatma[1*]

[1]Mathematics Study Program, Universitas Negeri Jakarta, Jakarta, Indonesia
[2]College of Interdisciplinary Studies, Zayed University, Abu Dhabi, United Arab Emirates

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Abstract

The notion of hyperstructures is a generalization of algebraic structures. This notion was first introduced by Marty in 1934. Hyperstructures have many applications, such as in biology, physics, cryptography, and chemistry. This paper focuses on the application of hyperstructures in chemistry, especially in chemical reactions. In 2022, Al-Tahan and Davvaz finalized the results of chemical hyperstructures for chemical elements that have four oxidation states. Motivated by this research, this paper aims to investigate algebraic hyperstructures in some elements that have five oxidation states, that is, neptunium, rubidium, and plutonium. Furthermore, the chemical interpretation of these chemical elements also is provided in this paper.

1 Introduction

Algebraic hyperstructures are a generalization of ordinary algebraic structures, first introduced by Marty in 1934 [1]. In algebraic hyperstructures, the concept of groups is generalized to hypergroups [2]. Numerous studies have explored the application of algebraic hyperstructures in physics, biology, and chemistry [3]. In physics, algebraic hyperstructures are applied in elementary particle physics (Leptons) [4, 5]. In Biology, they are applied in genetics, especially in the inheritance that is associated with fuzzy sets and intuitionistic fuzzy sets [6–9]. In this paper, we focus on the application of algebraic hyperstructures in the field of chemistry, specifically in chemical reactions [10–12]. Several studies have investigated algebraic hyperstructures in chemical elements [13–16]. The results are generalized for elements that have three oxidation states or four oxidation states [17, 18]. Algebraic hyperstructures can also be used to identify the types of hyperstructures present in electrochemical cell reactions [19]. Furthermore, chemical

*Corresponding author
E-mail addresses: fakhry_asad@yahoo.com (F. A. Agusfrianto), mariam.elhariri@gmail.com (M. Hariri), yudi_mahatma@unj.ac.id (Y. Mahatma)
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Table 1: $(G, \otimes)$.

$\otimes$	p	q	r
p	p	$\{p,q\}$	G
q	$\{p,q\}$	q	$\{q,r\}$
r	G	$\{q,r\}$	q

reactions that occur in everyday life, such as ozone layer depletion reactions and salt formation reactions, can also be investigated for the types of algebraic hyperstructures contained in these reactions [20–23].
Motivated by the generalization of algebraic hyperstructures for elements that have three and four oxidation states [17, 18], this paper aims to investigate the algebraic hyperstructures found in several elements that have five oxidation states, namely neptunium (Np), rubidium (Ru), and plutonium (Pu).

2 Preliminaries

In this section, we recall the basic concepts of algebraic hyperstructures that are used in this study. Let $\otimes : K \times K \to P^*(K)$ where $P^*(K)$ is the all-non-empty subset of K. The map $\otimes$ is called hyperoperation in K.

- If K is equipped with a hyperoperation $\otimes$, denoted by $(K, \otimes)$, then the mathematical system $(K, \otimes)$ is called a hypergroupoid [2]. Based on this definition, if X and Y are nonempty subsets of K, then for every $x \in H$, we denote:

$$X \otimes Y = \bigcup_{x \in X, y \in Y} x \otimes y,\ x \otimes X = \{x\} \otimes X, \text{ and } X \otimes x = X \otimes \{x\}.$$

- If the hyperoperation $\otimes$ is equipped with associative properties, namely for every $x, y, z \in K$, $(x \otimes y) \otimes z = x \otimes (y \otimes z)$, then the hypergroupoid $(K, \otimes)$ is called a semihypergroup.
- If the hypergroupoid $(K, \otimes)$ is equipped with reproduction axiom, i.e., for every $k \in K$, $k \otimes K = K \otimes k = K$, then it is called a quasihypergroup.
- If the hypergroupoid $(K, \otimes)$ is a semihypergroup and a quasihypergroup, then it is called a hypergroup [2].

Here is a simple example to clarify the definition.

Example 2.1. Given the set $G = \{p, q, r\}$. Define an operation "$\otimes$" as in Table 1. Then, $(G, \otimes)$ is a commutative hypergroup.

On the other hand, by Vougioklis the concept of algebraic hyperstructures is generalized into a concept called "H_v - structures" [24]. In H_v - structures, the weak concept is used to define its structure. Let K be the non-empty set and $\otimes : K \times K \to P^*(K)$ is a hyperoperation. The operation "$\otimes$" in K is called weak associative if for every $x, y, z \in K$, , $x \otimes (y \otimes z) \cap (x \otimes y) \otimes z \neq \emptyset$. The hypergroupoid $(K, \otimes)$ are called H_v - semihypergroups if weak associative properties are fulfilled. Furthermore, if the hypergroupoid $(K, \otimes)$ is equipped with reproduction axiom, then it is called H_v - group. To clarify this definition, consider the next Example 2.2.

Table 2: $(K, \otimes^*)$.

$\otimes^*$	p	q	r	s	t
p	p	$\{p,t\}$	$\{p,t\}$	$\{p,t\}$	$\{p,t\}$
q	$\{p,t\}$	$\{p,q\}$	p	p	p
r	$\{p,t\}$	p	$\{p,q\}$	p	p
s	$\{p,t\}$	p	p	$\{p,t\}$	p
t	$\{p,t\}$	p	$\{p,t\}$	$\{p,t\}$	$\{p,t\}$

Example 2.2. Given a set $K = \{p, q, r, s, t\}$. Define an operation $\otimes^*$ as in Table 2. Then, $(K, \otimes^*)$ is a commutative H_v- semigroup.

3 Main results

3.1 Chemical hyperstructures for neptunium

Neptunium, symbolized by Np, is an element located in the actinide series. This element has an atomic number of 93. The benefit of this element is that its isotopes, namely ^{238}Np and ^{239}Np, have a short half-life, so they are useful for radioactive tracers or research on basic chemistry. The Latimer diagram for Neptunium is given as follows:

$$NpO_2(OH)_2 \rightarrow_{0.6} NpO_2(OH)_2 \rightarrow_{0.3} Np(OH)_4 \rightarrow_{-2.1} Np(OH)_3 \rightarrow_{-2.23} Np.$$

This can be simplified to:

$$Np^{6+} \rightarrow_{0.6V} Np^{5+} \rightarrow_{0.3V} Np^{4+} \rightarrow_{-2.1V} Np^{3+} \rightarrow_{-2.23V} Np.$$

Next, we determine standard reduction potentials from chemical reactions that do not yet have a reduction potential.

$$\begin{aligned} Np^{6+} &\rightarrow Np^{4+}, E = 0.45, \\ Np^{6+} &\rightarrow Np^{3+}, E = -0.4, \\ Np^{6+} &\rightarrow Np, E = -1.315, \\ Np^{5+} &\rightarrow Np^{3+}, E = -0.9, \\ Np^{5+} &\rightarrow Np, E = -1.698, \\ Np^{4+} &\rightarrow Np, E = -2.1975. \end{aligned}$$

Let $X = \{Np^{6+}, Np^{5+}, Np^{4+}, Np^{3+}, Np\}$ and $\boxplus$ be reactions between two elements in X that are spontaneous and most positive. The whole possible spontaneous reaction of two elements in X is given as follows:

$$\begin{aligned} Np^{6+} + Np^{6+} &\rightarrow Np^{6+} + Np^{6+}[0], \\ Np^{6+} + Np^{5+} &\rightarrow Np^{6+} + Np^{5+}[0], \\ Np^{6+} + Np^{4+} &\rightarrow \begin{cases} Np^{6+} + Np^{4+}[0], \\ Np^{5+} + Np^{5+}[0.3], \\ Np^{5+} + Np^{5+}[0.15], \end{cases} \end{aligned}$$

$$Np^{6+} + Np^{3+} \rightarrow \begin{cases} Np^{6+} + Np^{3+}[0], \\ Np^{5+} + Np^{6+}[1.0], \\ Np^{5+} + Np^{5+}[1.2], \\ Np^{5+} + Np^{4+}[2.7], \\ Np^{4+} + Np^{6+}[0.85], \\ Np^{3+} + Np^{5+}[0.5]. \end{cases}$$

$$Np^{6+} + Np \rightarrow \begin{cases} Np^{6+} + Np[0], \\ Np^{5+} + Np^{6+}[1.915], \\ Np^{5+} + Np^{5+}[2.298], \\ Np^{5+} + Np^{4+}[2.7975], \\ Np^{5+} + Np^{3+}[2.83], \\ Np^{4+} + Np^{6+}[1.765], \\ Np^{4+} + Np^{4+}[2.6475], \\ Np^{3+} + Np^{6+}[0.915], \\ Np^{3+} + Np^{4+}[1.7], \\ Np^{3+} + Np^{3+}[1.83], \\ Np + Np^{5+}[0.383], \\ Np + Np^{4+}[0.8825], \\ Np + Np^{3+}[0.915], \end{cases}$$

$$Np^{5+} + Np^{5+} \rightarrow Np^{5+} + Np^{5+}[0],$$

$$Np^{5+} + Np^{4+} \rightarrow Np^{5+} + Np^{4+}[0],$$

$$Np^{5+} + Np^{3+} \rightarrow \begin{cases} Np^{5+} + Np^{3+}[0], \\ Np^{4+} + Np^{6+}[0.7], \\ Np^{4+} + Np^{5+}[1.2], \\ Np^{4+} + Np^{4+}[2.4], \\ Np^{3+} + Np^{4+}[1.2], \\ Np + Np^{4+}[0.402], \end{cases}$$

$$Np^{5+} + Np \rightarrow \begin{cases} Np^{5+} + Np[0], \\ Np^{4+} + Np^{6+}[1.615], \\ Np^{4+} + Np^{5+}[1.998], \\ Np^{4+} + Np^{4+}[2.4975], \\ Np^{4+} + Np^{3+}[2.53], \\ Np^{3+} + Np^{6+}[0.415], \\ Np^{3+} + Np^{5+}[0.798], \\ Np^{3+} + Np^{3+}[1.23], \\ Np + Np^{4+}[0.4995], \\ Np + Np^{3+}[0.532], \end{cases}$$

Table 3: $(X, \boxplus)$.

$\boxplus$	Np^{6+}	Np^{5+}	Np^{4+}	Np^{3+}	Np
Np^{6+}	Np^{6+}	$\{Np^{6+}, Np^{5+}\}$	Np^{5+}	$\{Np^{5+}, Np^{4+}\}$	$\{Np^{5+}, Np^{3+}\}$
Np^{5+}	$\{Np^{6+}, Np^{5+}\}$	Np^{5+}	$\{Np^{5+}, Np^{4+}\}$	Np^{4+}	$\{Np^{4+}, Np^{3+}\}$
Np^{4+}	Np^{5+}	$\{Np^{5+}, Np^{4+}\}$	Np^{4+}	$\{Np^{4+}, Np^{3+}\}$	Np^{3+}
Np^{3+}	$\{Np^{5+}, Np^{4+}\}$	Np^{4+}	$\{Np^{4+}, Np^{3+}\}$	Np^{3+}	Np
Np	$\{Np^{5+}, Np^{3+}\}$	$\{Np^{4+}, Np^{3+}\}$	Np^{4+}	$\{Np^{3+}, Np\}$	Np

Table 4: General Form of Table 3.

$\boxplus$	t	u	v	w	x
t	t	$\{t, u\}$	u	$\{u, v\}$	$\{u, w\}$
u	$\{t, u\}$	u	$\{u, v\}$	v	$\{v, w\}$
v	u	$\{u, v\}$	v	$\{v, w\}$	w
w	$\{u, v\}$	v	$\{v, w\}$	w	$\{w, x\}$
x	$\{u, w\}$	$\{v, w\}$	w	$\{w, x\}$	x

$$
\begin{gathered}
Np^{4+} + Np^{4+} \to Np^{4+} + Np^{4+}[0], \\
Np^{4+} + Np^{3+} \to Np^{4+} + Np^{3+}[0], \\
Np^{4+} + Np \to \begin{cases} Np^{4+} + Np[0], \\ Np^{3+} + Np^{4+}[0.0975], \\ Np^{3+} + Np^{3+}[0.13], \end{cases} \\
Np^{3+} + Np^{3+} \to Np^{3+} + Np^{3+}[0], \\
Np^{3+} + Np \to Np^{3+} + Np[0], \\
Np + Np \to Np + Np[0].
\end{gathered}
$$

Then, we can write $(X, \boxplus)$ as in Table 3. Let $Np^{6+} = t$, $Np^{5+} = u$, $Np^{4+} = v$, $Np^{3+} = w$, and $Np = x$. In general, Table 3 can be written as Table 4.

Theorem 3.1. *$(X, \boxplus)$ is a H_v - semigroup.*

Proof. It is clear that "$\boxplus$" is a hyperoperation of X. To prove that $(X, \boxplus)$ is H_v - semigroup, we should prove that $\boxplus$ is a weak associative i.e., for every $a, b, c \in X$, $[a \boxplus (b \boxplus c)] \cap [(a \boxplus b) \boxplus c] \neq \emptyset$.
Case 1. $a = t$. It is obvious that $t \in [t \boxplus (b \boxplus c)] \cap [(t \boxplus b) \boxplus c]$.
Case 2. $a = u$. For $a = u$, if $b = w$ and $c = x$, we get $[u \boxplus (w \boxplus x)] \cap [(u \boxplus w) \cap x] = w$. Otherwise, $u \in [u \boxplus (b \boxplus c)] \cap [(u \boxplus b) \cap c]$.
Case 3. $a = v$. For $a = v$, if $b = c = t$ and $b = c = x$, we get $[v \boxplus (t \boxplus t)] \cap [(v \boxplus t) \boxplus t] = u$ and $[v \boxplus (x \boxplus x)] \cap [(v \boxplus x) \boxplus x] = w$ respectively. Otherwise, $v \in [v \boxplus (b \boxplus c)] \cap [(v \boxplus b) \cap c]$.
Case 4. $a = w$. For $a = w$, if $b = c = t$ and $b = c = u$, we get $[w \boxplus (t \boxplus t)] \cap [(w \boxplus t) \boxplus t] = u$ and $[w \boxplus (u \boxplus u)] \cap [(w \boxplus u) \boxplus u] = v$ respectively. Otherwise, $w \in [w \boxplus (b \boxplus c)] \cap [(w \boxplus b) \boxplus c]$.
Case 5. $a = x$. For $a = x$, if $b = c = x$, we get $[x \boxplus (x \boxplus x)] \cap [(x \boxplus x) \boxplus x] = x$. Otherwise, $w \in [x \boxplus (b \boxplus c)] \cap [(x \boxplus b) \boxplus c]$.
Thus, $(X, \boxplus)$ is a H_v - semigroup. ■

Remark 1. $(X, \boxplus)$ is not a semihypergroup since we have $[v \boxplus (t \boxplus t)] = u \neq [(v \boxplus t) \boxplus t] = \{u, v\}$.

Table 5: $(X, \boxplus^*)$.

$\boxplus*$	t	u	v	w	x
t	t	$\{t,u\}$	$\{t,u,v\}$	$\{t,u,v,w\}$	X
u	$\{t,u\}$	u	$\{u,v\}$	X	X
v	$\{t,u,v\}$	$\{u,v\}$	v	$\{v,w\}$	$\{v,w,x\}$
w	$\{t,u,v,w\}$	X	$\{v,w\}$	w	$\{w,x\}$
x	X	X	$\{v,w,x\}$	$\{w,x\}$	x

Furthermore, suppose that "$\boxplus^*$" is defined as the entire spontaneous reaction that occurs between two elements in X, then Table 5 is obtained.

Theorem 3.2. *$(\{t,u\}, \boxplus^*)$, $(\{u,v\}, \boxplus^*)$, $(\{v,w\}, \boxplus^*)$, and $(\{w,x\}, \boxplus^*)$ are hypergroups.*

Proof. The proof is clear. ■

Theorem 3.3. *$(X, \boxplus^*)$ is a semihypergroup.*

Proof. Since $(\{t,u\}, \boxplus^*)$, $(\{u,v\}, \boxplus^*)$, $(\{v,w\}, \boxplus^*)$, and $(\{w,x\}, \boxplus^*)$ are hypergroups, then it is sufficient to prove the following conditions:

1. $a \boxplus^* (b \boxplus^* c) = (a \boxplus^* b) \boxplus^* c$ for $a \in \{t,u\}$ and $\{b,c\} \nsubseteq \{t,u\}$.
2. $a \boxplus^* (b \boxplus^* c) = (a \boxplus^* b) \boxplus^* c$ for $a \in \{v,w\}$ and $\{b,c\} \nsubseteq \{v,w\}$.
3. $e \boxplus^* (b \boxplus^* c) = (e \boxplus^* b) \boxplus^* c$ for $\{b,c\} \nsubseteq \{w,x\}$.

$$t \boxplus^* (v \boxplus^* c) = \begin{cases} \{t,u,v\}, c = t,u,v, \\ \{t,u,v,w\}, c = w, \\ X, c = x, \end{cases} = (t \boxplus^* v) \boxplus^* c,$$

$$\begin{aligned}
t \boxplus^* (w \boxplus^* c) &= \begin{cases} \{t,u,v,w\}, c = a,c,d, \\ X, c = b,e, \end{cases} = (t \boxplus^* w) \boxplus^* c, \\
t \boxplus^* (x \boxplus^* c) &= (t \boxplus^* x) \boxplus^* c = X, \\
u \boxplus^* (x \boxplus^* c) &= \begin{cases} \{t,u,v\}, z = t, \\ \{u,v\}, z = u,v, \\ X, z = w,x, \end{cases} = (t \boxplus^* x) \boxplus^* c, \\
t \boxplus^* (w \boxplus^* c) &= (t \boxplus^* w) \boxplus^* c = X, \\
t \boxplus^* (x \boxplus^* c) &= (t \boxplus^* x) \boxplus^* c = X, \\
v \boxplus^* (t \boxplus^* c) &= \begin{cases} \{t,u,v\}, c = t,u,v, \\ \{t,u,v,w\}, c = d, \\ X, c = e, \end{cases} = (v \boxplus^* t) \boxplus^* c,
\end{aligned}$$

$$v \boxplus^* (u \boxplus^* c) = \begin{cases} \{t, u, v\}, c = t, \\ \{u, v\}, c = u, v, \\ X, c = d, e, \end{cases} = (v \boxplus^* u) \boxplus^* c,$$

$$v \boxplus^* (x \boxplus^* c) = \begin{cases} \{v, w, x\}, c = v, w, x, \\ X, c = a, b, \end{cases} = (v \boxplus^* x) \boxplus^* c,$$

$$w \boxplus^* (w \boxplus^* c) = \begin{cases} X, c = t, u, \\ \{v, w, x\}, c = v, \\ \{w, x\}, c = v, w, \end{cases} = (w \boxplus^* w) \boxplus^* c,$$

$$e \boxplus^* (b \boxplus^* c) = (e \boxplus^* b) \boxplus^* c = X.$$

Therefore, $(X, \boxplus^*)$ is a semihypergroup. ■

Theorem 3.4. $(X, \boxplus^*)$ *is a quasi-hypergroup.*

Proof. It is obvious because all rows and columns contain X. ■

Corollary 3.5. $(X, \boxplus^*)$ *is a hypergroup.*

3.2 Chemical hyperstructures for rubidium

Rubidium, symbolized by Ru, is an element that has an atomic number of 37, period 5, and is located in the IA group. Ru is a metallic element that easily loses electrons in its outermost layer. Ru has applications in high-tech fields, such as MHD power generation, ion propulsion engines, and thermionic power conversion. The Latimer diagram for rubidium is given as follows:

$$RuO_4 \rightarrow_{1.00} RuO_4^- \rightarrow_{0.59} RuO_4^{2-} \rightarrow_{0.35} RuO_2 \rightarrow_{-0.15} Ru.$$

This can be simplified to:

$$Ru^{8+} \rightarrow_{1.00} Ru^{7+} \rightarrow_{0.59} Ru^{6+} \rightarrow_{0.35} Ru^{4+} \rightarrow_{-0.15} Ru.$$

Furthermore, we determine standard potential reductions from chemical reactions that do not yet have standard potential reductions. In the same way, when looking for standard potential reduction from Rubidium, the following results are obtained:

$$\begin{aligned} Ru^{8+} \rightarrow Ru^{6+}, &\quad E = 0.759, \\ Ru^{8+} \rightarrow Ru^{4+}, &\quad E = 0.5725, \\ Ru^{8+} \rightarrow Ru, &\quad E = 0.2113, \\ Ru^{7+} \rightarrow Ru^{4+}, &\quad E = 0.43, \\ Ru^{7+} \rightarrow Ru, &\quad E = 0.0986, \\ Ru^{6+} \rightarrow Ru, &\quad E = 0.1667. \end{aligned}$$

Suppose that $K = \{Ru^{8+}, Ru^{7+}, Ru^{6+}, Ru^{4+}, Ru\}$ and define $\odot$ as a spontaneous reaction that occurs between two elements in K. The whole spontaneous reaction that occurs between two elements in K is obtained as follows:

$$Ru^{8+} + Ru^{8+} \rightarrow Ru^{8+} + Ru^{8+}[0],$$

$$Ru^{8+} + Ru^{7+} \to Ru^{8+} + Ru^{7+}[0],$$

$$Ru^{8+} + Ru^{6+} \to \begin{cases} Ru^{8+} + Ru^{6+}[0], \\ Ru^{7+} + Ru^{8+}[0], \\ Ru^{7+} + Ru^{7+}[0.41], \end{cases}$$

$$Ru^{8+} + Ru^{4+} \to \begin{cases} Ru^{8+} + Ru^{4+}[0], \\ Ru^{7+} + Ru^{8+}[0.4275], \\ Ru^{7+} + Ru^{7+}[0.57], \\ Ru^{7+} + Ru^{6+}[0.65], \\ Ru^{6+} + Ru^{6+}[0.409], \\ Ru^{4+} + Ru^{7+}[0.1425]. \end{cases}$$

$$Ru^{8+} + Ru \to \begin{cases} Ru^{8+} + Ru[0], \\ Ru^{7+} + Ru^{8+}[0.7887], \\ Ru^{7+} + Ru^{7+}[0.9014], \\ Ru^{7+} + Ru^{4+}[1.15], \\ Ru^{6+} + Ru^{8+}[0.5477], \\ Ru^{6+} + Ru^{7+}[0.6604], \\ Ru^{6+} + Ru^{6+}[0.5923], \\ Ru^{4+} + Ru^{8+}[0.3612], \\ Ru^{4+} + Ru^{7+}[0.4739], \\ Ru^{4+} + Ru^{6+}[0.4058], \\ Ru^{4+} + Ru^{4+}[0.7225], \\ Ru + Ru^{7+}[0.1127], \\ Ru + Ru^{6+}[0.0446], \\ Ru + Ru^{4+}[0.3613]. \end{cases}$$

$$Ru^{7+} + Ru^{7+} \to Ru^{7+} + Ru^{7+}[0],$$

$$Ru^{7+} + Ru^{6+} \to Ru^{7+} + Ru^{6+}[0],$$

$$Ru^{7+} + Ru^{4+} \to \begin{cases} Ru^{7+} + Ru^{4+}[0], \\ Ru^{6+} + Ru^{8+}[0.0175], \\ Ru^{6+} + Ru^{7+}[0.16], \\ Ru^{6+} + Ru^{6+}[0.24], \\ Ru^{4+} + Ru^{6+}[0.08]. \end{cases}$$

$$Ru^{7+} + Ru \to \begin{cases} Ru^{7+} + Ru[0], \\ Ru^{6+} + Ru^{8+}[0.3787], \\ Ru^{6+} + Ru^{7+}[0.4914], \\ Ru^{6+} + Ru^{6+}[0.4233], \\ Ru^{6+} + Ru^{4+}[0.74], \\ Ru^{4+} + Ru^{7+}[0.3314], \\ Ru^{4+} + Ru^{4+}[0.58], \\ Ru + Ru^{4+}[0.2468]. \end{cases}$$

Table 6: $(K, \odot)$.

$\odot$	t	u	v	w	x
t	t	$\{t,u\}$	u	$\{u,v\}$	$\{u,w\}$
u	$\{t,u\}$	u	$\{u,v\}$	v	$\{v,w\}$
v	u	$\{u,v\}$	v	$\{v,w\}$	w
w	$\{u,v\}$	v	$\{v,w\}$	w	$\{w,x\}$
x	$\{u,w\}$	$\{v,w\}$	w	$\{w,x\}$	x

Table 7: $(K, \odot^*)$.

$\odot*$	t	u	v	w	x
t	t	$\{t,u\}$	$\{t,u,v\}$	$\{t,u,v,w\}$	K
u	$\{t,u\}$	u	$\{u,v\}$	K	K
v	$\{t,u,v\}$	$\{u,v\}$	v	$\{v,w\}$	$\{v,w,x\}$
w	$\{t,u,v,w\}$	X	$\{v,w\}$	w	$\{w,x\}$
x	K	K	$\{v,w,x\}$	$\{w,x\}$	x

$$Ru^{6+} + Ru^{6+} \to Ru^{6+} + Ru^{6+}[0],$$
$$Ru^{6+} + Ru^{4+} \to Ru^{6+} + Ru^{4+}[0],$$
$$Ru^{6+} + Ru \to \begin{cases} Ru^{6+} + Ru[0], \\ Ru^{4+} + Ru^{8+}[0.1387], \\ Ru^{4+} + Ru^{7+}[0.2514], \\ Ru^{4+} + Ru^{6+}[0.1883], \\ Ru^{4+} + Ru^{4+}[0.50], \\ Ru + Ru^{7+}[0.0681], \\ Ru + Ru^{4+}[0.3167]. \end{cases}$$
$$Ru^{4+} + Ru^{4+} \to Ru^{4+} + Ru[0],$$

$$Ru^{4+} + Ru \to Ru^{4+} + Ru[0],$$
$$Ru + Ru \to Ru + Ru[0].$$

Let $Ru^{8+} = t$, $Ru^{7+} = u$, $Ru^{6+} = v$, $Ru^{4+} = w$, and $Ru = x$. Then, we have Table 6.

Theorem 3.6. *$(K, \odot)$ is a H_v - semigroup.*

Proof. It is obvious since $(K, \odot) \cong (X, \boxplus)$ ■

Now, suppose that "$\odot^*$" is defined as the entire spontaneous reaction that occurs between two elements in K, then Table 7 is obtained.

Theorem 3.7. *$(K, \odot^*)$ is a hypergroup.*

3.3 Chemical hyperstructures for plutonium

Plutonium, symbolized by Pu, is a chemical element that has an atomic number of 94 and belongs to the actinide metal type. Plutonium is a radioactive element whose isotopes can be useful as explosives. The Latimer diagram of plutonium is as follows:

$$PuO_2(OH)_2 \to_{0.3} PuO_2OH \to_{0.9} Pu(OH)_4 \to_{-1.4} Pu(OH)_3 \to_{-2.46} Pu.$$

This can be simplified to:

$$Pu^{6+} \to_{0.3} Pu^{5+} \to_{0.9} Pu^{4+} \to_{-1.4} Pu^{3+} \to_{-2.46} Pu.$$

Next, we determined standard potential reductions from chemical reactions that do not yet have a standard potential reduction. In the same way when looking for standard potential reduction from neptunium, the following results are obtained:

$$\begin{aligned} Pu^{6+} \to Pu^{4+}, \quad E = 0.6, \\ Pu^{6+} \to Pu^{3+}, \quad E = -0.067, \\ Pu^{6+} \to Pu, \quad E = -1,263, \\ Pu^{5+} \to Pu^{3+}, \quad E = -0.25, \\ Pu^{5+} \to Pu, \quad E = -1.576, \\ Pu^{4+} \to Pu, \quad E = -2.195. \end{aligned}$$

Suppose that $M = \{Pu^{6+}, Pu^{5+}, Pu^{4+}, Pu^{3+}, Pu\}$ and define a hyperoperation $\boxdot$ as a most positive spontaneous reaction that occurs between two elements in M. The whole spontaneous reaction that occurs between two elements in M is obtained as follows:

$$Pu^{6+} + Pu^{6+} \to Pu^{6+} + Pu^{6+}[0],$$

$$Pu^{6+} + Pu^{5+} \to \begin{cases} Pu^{6+} + Pu^{5+}[0], \\ Pu^{4+} + Pu^{6+}[0.3]. \end{cases}$$

$$Pu^{6+} + Pu^{4+} \to Pu^{6+} + Pu^{4+}[0],$$

$$Pu^{6+} + Pu^{3+} \to \begin{cases} Pu^{6+} + Pu^{3+}[0], \\ Pu^{5+} + Pu^{6+}[0.367], \\ Pu^{5+} + Pu^{5+}[0.55], \\ Pu^{4+} + Pu^{6+}[0.667], \\ Pu^{4+} + Pu^{5+}[0.85], \\ Pu^{4+} + Pu^{4+}[2.0], \\ Pu^{3+} + Pu^{5+}[0.183], \\ Pu^{3+} + Pu^{5+}[1.333], \\ Pu + Pu^{4+}[0.137]. \end{cases}$$

$$Pu^{6+} + Pu \to \begin{cases} Pu^{6+} + Pu[0], \\ Pu^{5+} + Pu^{6+}[1.563], \\ Pu^{5+} + Pu^{5+}[1.876], \\ Pu^{5+} + Pu^{4+}[2.495], \\ Pu^{4+} + Pu^{6+}[1.863], \\ Pu^{4+} + Pu^{5+}[2.176], \\ Pu^{4+} + Pu^{4+}[2.795], \\ Pu^{3+} + Pu^{6+}[1.2], \\ Pu^{3+} + Pu^{5+}[1.509], \\ Pu^{3+} + Pu^{4+}[2.128], \\ Pu^{3+} + Pu^{3+}[2.393], \\ Pu + Pu^{5+}[0.313], \\ Pu + Pu^{4+}[0.932], \\ Pu + Pu^{3+}[1.197]. \end{cases}$$

$$Pu^{5+} + Pu^{5+} \to \begin{cases} Pu^{5+} + Pu^{5+}[0], \\ Pu^{6+} + Pu^{4+}[0.6]. \end{cases}$$

$$Pu^{5+} + Pu^{4+} \to \begin{cases} Pu^{5+} + Pu^{4+}[0], \\ Pu^{4+} + Pu^{6+}[0.3]. \end{cases}$$

$$Pu^{5+} + Pu^{3+} \to \begin{cases} Pu^{5+} + Pu^{3+}[0], \\ Pu^{4+} + Pu^{6+}[0.967], \\ Pu^{4+} + Pu^{5+}[1.15], \\ Pu^{4+} + Pu^{4+}[2.3], \\ Pu^{3+} + Pu^{4+}[1.15]. \end{cases}$$

$$Pu^{5+} + Pu \to \begin{cases} Pu^{5+} + Pu[0], \\ Pu^{4+} + Pu^{6+}[2.163], \\ Pu^{4+} + Pu^{5+}[2.467], \\ Pu^{4+} + Pu^{4+}[3.095], \\ Pu^{4+} + Pu^{3+}[3.36], \\ Pu^{3+} + Pu^{6+}[1.013], \\ Pu^{3+} + Pu^{5+}[1.326], \\ Pu^{3+} + Pu^{4+}[1.945], \\ Pu^{3+} + Pu^{3+}[2.21]. \end{cases}$$

$$Pu^{4+} + Pu^{4+} \to Pu^{4+} + Pu^{4+}[0],$$

$$Pu^{4+} + Pu^{3+} \to Pu^{4+} + Pu^{3+}[0].$$

$$Pu^{4+} + Pu \to \begin{cases} Pu^{4+} + Pu[0], \\ Pu^{3+} + Pu^{5+}[0.176], \\ Pu^{3+} + Pu^{4+}[0.795], \\ Pu^{3+} + Pu^{3+}[1.06], \\ Pu + Pu^{3+}[0.265]. \end{cases}$$

Table 8: $(M, \boxdot)$.

$\boxdot$	a	b	c	d	e
a	a	$\{a, c\}$	$\{a, c\}$	c	c
b	$\{a, c\}$	$\{a, c\}$	$\{a, c\}$	c	$\{c, d\}$
c	$\{a, c\}$	$\{a, c\}$	c	$\{c, d\}$	d
d	c	c	$\{c, d\}$	d	$\{d, e\}$
e	c	$\{c, d\}$	d	$\{d, e\}$	e

Table 9: $(M, \boxdot^*)$.

$\boxdot^*$	a	b	c	d	e
a	a	$\{a, b, c\}$	$\{a, c\}$	M	M
b	$\{a, b, c\}$	$\{a, b, c\}$	$\{a, b, c\}$	$\{a, b, c, d\}$	M
c	$\{a, c\}$	$\{a, b, c\}$	c	$\{c, d\}$	$\{b, c, d, e\}$
d	M	$\{a, b, c, d\}$	$\{c, d\}$	d	$\{d, e\}$
e	M	M	$\{b, c, d, e\}$	$\{d, e\}$	e

$$Pu^{3+} + Pu^{3+} \to Pu^{3+} + Pu^{3+}[0],$$
$$Pu^{3+} + Pu \to Pu^{3+} + Pu[0],$$
$$Pu + Pu \to Pu + Pu[0].$$

Let $a = Pu^{6+}, b = Pu^{5+}, c = Pu^{4+}, d = Pu^{3+}, e = Pu$. Then, we have Table 8.

Theorem 3.8. *$(M, \boxdot)$ is a H_v - semigroup.*

Proof. It is clear that "$\boxdot$" is a hyperoperation of M. To prove that $(M, \boxdot)$ is a H_v - semigroup, we should prove that a hyperoperation $\boxdot$ is a weak associative i.e., for every $x, y, z \in M$, $[x \boxdot (y \boxdot z)] \cap [(x \boxdot y) \boxdot z] \neq \emptyset$.
Case 1 $x = a$. For $x = a$, if $y = z = a$, we get $[a \boxdot (a \boxdot a)] \cap [(a \boxdot a) \boxdot a] = \{a\} \neq \emptyset$. Otherwise, it is clear that $c \in [a \boxdot (y \boxdot z)] \cap [(a \boxdot y) \boxdot z]$.
Case 2 $x = b$. It is clear that $c \in [(b \boxdot (y \boxdot z)] \cap [(b \boxdot y) \boxdot z]$.
Case 3 $x = c$. For $x = c$, if $y = z = e$, we get $[c \boxdot (e \boxdot e)] \cap [(c \boxdot e) \boxdot e] = \{d\} \neq \emptyset$. Otherwise, it is clear that $c \in [c \boxdot (y \boxdot z)] \cap [(c \boxdot y) \boxdot z]$.
Case 4 $x = d$. For $x = d$, if $y = d$ and $z = e$, we get $[d \boxdot (d \boxdot e)] \cap [(d \boxdot d) \boxdot e] = \{d, e\} \neq \emptyset$. If $x = y = d$, we get $[d \boxdot (d \boxdot d)] \cap [(d \boxdot d) \boxdot d] = d \neq \emptyset$. Otherwise, it is clear that $c \in [d \boxdot (y \boxdot z)] \cap [(d \boxdot y) \boxdot z]$.
Case 5 $x = e$. For $x = e$, if $y = z = a$ and $y = z = e$, we get $[e \boxdot (a \boxdot a)] \cap [(e \boxdot a) \boxdot a] = c \neq \emptyset$ and $[e \boxdot (e \boxdot e)] \cap [(e \boxdot e) \boxdot e] = e \neq \emptyset$ respectively, Otherwise, it is clear that $d \in [e \boxdot (y \boxdot z)] \cap [(e \boxdot y) \boxdot z]$
Since $(M, \boxdot)$ is a weak associative, then $(M, \boxdot)$ is a H_v - semigroup. ■

Remark 2. $(M, \boxdot)$ is not a semihypergroup since $c \boxdot (e \boxdot e) = d \neq (c \boxdot e) \boxdot e = \{d, e\}$.
Now, define a hyperoperation $\boxdot^*$ as the entire spontaneous reaction that occurs between two elements in M. Then, Table 9 is obtained.

Theorem 3.9. *$(\{c, d\}, \boxdot)$ and $(\{d, e\}, \boxdot)$ is a hypergroup.*

Proof. Obvious. ■

Theorem 3.10. *$(M, \boxdot^*)$ is a quasi-hypergroup.*

Proof. It is obvious because all rows and columns contain M. ■

Theorem 3.11. *$(M, \boxdot^*)$ is a semihypergroup.*

Proof. Since $(\{c, d\}, \boxdot)$ and $(\{d, e\}, \boxdot)$ is a hypergroup, then it is enough to prove the following condition:

1. $x \boxdot^* (y \boxdot^* z) = (x \boxdot^* y) \boxdot^* z$ for $x \in \{c, d\}$ and $\{y, z\} \nsubseteq \{c, d\}$,
2. $a \boxdot^* (y \boxdot^* z) = (a \boxdot^* y) \boxdot^* z$,
3. $b \boxdot^* (y \boxdot^* z) = (b \boxdot^* y) \boxdot^* z$,
4. $e \boxdot^* (y \boxdot^* z) = (e \boxdot^* y) \boxdot^* z$ for $\{y, z\} \nsubseteq \{d, e\}$.

$$c \boxdot^* (a \boxdot^* z) = \begin{cases} \{a, c\}, z = a, c, \\ \{a, b, c\}, z = b, \\ M, z = d, e, \end{cases} = (c \boxdot^* a) \boxdot^* z,$$

$$c \boxdot^* (b \boxdot^* z) = \begin{cases} \{a, b, c\}, z = a, b, c, \\ \{a, b, c, d\}, z = d, \\ M, z = e, \end{cases} = (c \boxdot^* b) \boxdot^* z,$$

$$c \boxdot^* (e \boxdot^* z) = \begin{cases} \{b, c, d, e\}, z = d, e, \\ M, z = a, b, c, \end{cases} = (c \boxdot^* e) \boxdot^* z,$$

$$d \boxdot^* (y \boxdot^* z) = (d \boxdot^* y) \boxdot^* z = M,$$

$$a \boxdot^* (a \boxdot^* z) = \begin{cases} a, z = a, \\ \{a, b, c\}, z = b, \\ \{a, c\}, z = c, \\ M, z = d, e, \end{cases} = (a \boxdot^* a) \boxdot^* z,$$

$$a \boxdot^* (b \boxdot^* z) = \begin{cases} \{a, b, c\}, z = a, b, c, \\ M, z = d, e, \end{cases} = (a \boxdot^* b) \boxdot^* z,$$

$$a \boxdot^* (c \boxdot^* z) = \begin{cases} \{a, c\}, z = a, c, \\ \{a, b, c\}, z = b, \\ M, z = d, e, \end{cases} = (a \boxdot^* c) \boxdot^* z,$$

$$a \boxdot^* (\{d, e\} \boxdot^* z) = (a \boxdot^* \{d, e\}) \boxdot^* z,$$

$$b \boxdot^* (\{a, b, c\} \boxdot^* z) = \begin{cases} \{a, b, c\}, z = a, b, c, \\ \{a, b, c, d\}, z = d, \\ M, z = e, \end{cases} = (b \boxdot^* \{a, b, c\}) \boxdot^* z,$$

$$b \boxdot^* (d \boxdot^* z) = \begin{cases} \{a, b, c, d\}, z = b, c, d, \\ M, z = a, e, \end{cases} \quad = (b \boxdot^* d) \boxdot^* z,$$
$$b \boxdot^* (e \boxdot^* z) = (b \boxdot^* e) \boxdot^* z = M,$$
$$e \boxdot^* (y \boxdot^* z) = (e \boxdot^* y) \boxdot^* z = M.$$

Thus, $(M, \boxdot^*)$ is a semihypergroup. ■

Corollary 3.12. $(M, \boxdot^*)$ *is a hypergroup.*

4 Chemical interpretation

In this section, we give chemical interpretations of the hyperstructures that we have obtained for redox reactions in neptunium, rubidium, and plutonium.

Remark 3. Based on Table 5, we can conclude that Np^{4+} is the most abundant in nature and Np is the least abundant in nature.

Remark 4. Based on Table 7, we can conclude that Ru^{4+} is the most abundant in nature and Ru is the least abundant in nature.

Remark 5. Based on Table 9, we can conclude that Pu^{4+} is the most abundant in nature and Pu is the least abundant in nature.

Remark 6. The hyperstructures for neptunium and rubidium are isomorphic. This is due to the similarity in the condition of the Latimer diagram. That is, the Latimer diagrams of neptunium and rubidium have potential conditions that decrease as their oxidation states decrease.

Remark 7. The hyperstructure for plutonium is not isomorphic to the hyperstructures of neptunium and rubidium because the hyperstructure for plutonium has four idempotent elements, and the hyperstructures for neptunium and rubidium have five idempotent elements. This occurs because of the difference in the conditions of the Latimer diagram between plutonium, neptunium, and rubidium. The potential conditions on the plutonium Latimer diagram increase and then decrease, unlike the conditions on the rubidium and neptunium Latimer diagrams which decrease as their oxidation states decrease.

5 Concluding remarks

Based on the explanation above, we have determined the types of hyperstructures for some elements that have five oxidation states. These elements include neptunium, rubidium, and plutonium. We also find that the hyperstructures of plutonium are not isomorphic to the hyperstructures of neptunium and rubidium. For future research, these results can be generalized specifically to elements that have five oxidation states.

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References

[1] F. Marty, *Sur une generalization de la notion de group*, In 8th Congress Math. Scandenaves, Stockholm. (1934), 45 – 49.

[2] B. Davvaz and V. Leoreanu-Fotea, *Hypergroup Theory*, World Scientific, 2022.

[3] B. Davvaz and T. Vougiouklis, *A Walk Through Weak Hyperstructures H_v- Structures*, World Scientific Publishing Co. Pte. Ltd., Hackensack, NJ, 2019.

[4] A. Dehghan Nezhad, S. Moosavi Nejad, M. Nadjafikhah and B. Davvaz, A physical example of algebraic hyperstructures : Leptons, *Indian. J. Phys.* **86** (2012) 1027 – 1032, https://doi.org/10.1007/s12648-012-0151-x.

[5] B. Davvaz, A. Dehghan Nezhad and S. M. Moosavi Nejad , Algebraic hyperstructures of observable elementary particles including the higgs Boson, *Proc. Natl. Acad. Sci., India, Sect. A Phys. Sci.* **90** (2020) 169–176, https://doi.org/10.1007/s40010-018-0553-z.

[6] M. Al-Tahan and B. Davvaz, Algebraic hyperstructures associated to biological inheritance, *Math. Biosci.* **285** (2017) 112–118, https://doi.org/10.1016/j.mbs.2017.01.002.

[7] M. Al-Tahan and B. Davvaz, N-ary hyperstructures associated to the genotypes of F_2-offspring, *Int. J. Biomath.* **10** (2017) #1750118, https://doi.org/10.1142/S1793524517501182.

[8] M. Al-Tahan and B. Davvaz, Fuzzy subsets of the phenotypes of F_2-offspring, *Ser. Math. Inform.* **34** (2019) 709 – 727, https://doi.org/10.22190/FUMI1904709A.

[9] M. Al-Tahan and B. Davvaz, A new relationship between fuzzy sets and genetics, *J. Classification* **36** (2019) 494–512, https://doi.org/10.1007/s00357-018-9276-8.

[10] B. Davvaz, A. Dehghan Nezhad and A. Benvidi, Chemical hyperalgebra : dismutation reactions, *MATCH Commun. Math. Comput. Chem.* **67** (2012) 55 – 63.

[11] B. Davvaz, A. Dehghan Nezhad and M. Mazloum-Ardakani, Chemical hyperalgebra : redox reactions, *MATCH Commun. Math. Comput. Chem.* **71** (2014) 323 – 331.

[12] B. Davvaz, Weak Algebraic Hyperstructures as A Model for Interpretation of Chemical Reactions, *Iranian J. Math. Chem.* **7** (2016) 267 – 283, https://doi.org/10.22052/IJMC.2016.13975.

[13] K. M. Chun, Chemical hyperstructures of chemical reaction for Iron and Indium, *J. Chung. Math. Soc.* **27** (2014) 319 – 325, https://doi.org/ 10.14403/jcms.2014.27.2.319.

[14] K. M. Chun, Chemical hyperstructures of chemical reactions for Titanium, *J. Chung. Math. Soc.* **30** (2017) 459 – 466, https://doi.org/10.14403/jcms.2017.30.4.459.

[15] S. C. Chung, Chemical hyperstructrues for Vanadium, *J. Chung. Math. Soc.* **27** (2014) 309 – 317, https://doi.org/10.14403/jcms.2014.27.2.309.

[16] M. Al-Tahan and B. Davvaz, Chemical hyperstructures for astatine, tellurium, and for bismuth, *Bull. Comput. Appl. Math.* **7** (2019) 9 – 25.

[17] M. Al-Tahan and B. Davvaz, Chemical hyperstructures for elements with four oxidation states, *Iranian J. Math. Chem.* **13** (2022) 85–97, https://doi.org/10.22052/IJMC.2022.246174.1615.

[18] S. C. Chung, K. M. Chun, N. J. Kim, S. Y. Jeong, H. Sim, J. Lee and H. Maeng, Chemical hyperalgebra for three oxidation states of elements, *MATCH Commun. Math. Comput. Chem.* **72** (2014) 389 – 402.

[19] M. Al-Tahan and B. Davvaz, Weak chemical hyperstructures associated to electrochemical cells, *Iranian J. Math. Chem.* **9** (2018) 65 – 75, https://doi.org/10.22052/IJMC.2017.88790.1294.

[20] D. Heidari, D. Mazaheri and B. Davvaz, Chemical salt reactions as algebraic hyperstructures, *Iranian J. Math. Chem.* **10** (2019) 93 – 102, https://doi.org/10.22052/IJMC.2018.114473.1339.

[21] S. C. Chung, Chemical hyperstructures for ozone depletion, *J. Chungcheong Math. Soc.* **32** (2019) 491 – 508.

[22] S. C. Chung and K. M. Chun, Chemical hyperstructures for stratospheric ozone depletion, *J. Chungcheong Math. Soc.* **33** (2020) 469 – 487, https://doi.org/10.14403/jcms.2020.33.4.469.

[23] M. Al-Tahan and B. Davvaz, On quasi-ordering hypergroups, ordered hyperstructures and their applications in genetics, *Math. Interdisc. Res.* **7** (2022) 1–19.

[24] T. Vougiouklis, H_v-groups defined on the same set, *Discrete Math.* **155** (1996) 259 – 265, https://doi.org/10.1016/0012-365X(94)00390-5.