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Research Paper

Investigation of the Partition Dimension in Chemical Networks and its Application in Chemistry

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Abstract

Partition dimension problems involve dividing a graph's vertex set into a minimum number of disjoint sets so that each vertex is different with respect to the representation from each disjoint set. As a result of the development of this method, a number of applications have arisen in a number of fields such as drug design, navigation of robots, pattern recognition, and image processing. In this paper, we have calculated the partition dimension of oxide and zigzag benzenoid networks, and the subdivision of benzenoid hydrocarbon and triangular benzene networks.

1 Introduction

One of the important areas of mathematics is graph theory, whose mathematical ideas have been extensively applied to computer science research areas like networking, data mining, and image capturing [1]. The field has been greatly supported by graph theory, and its ideas have given rise to various exciting real-world applications throughout the Internet of Things, as well as for routing, consumer analysis, scheduling, and fraud detection. The graphs' metric dimension is one of the distance-related characteristics that are dealt with in graph theory. Slater [2] and Harary et al. [3] independently proposed the concept of the metric dimension of a graph. It has been presented by Chartrand et al. [4] to generalize the graph's metric dimension as a partition dimension. In the metric dimension, the distance between vertices is calculated, while in the partition dimension, the distance between a vertex and a set containing vertices is calculated. Determining a graph's metric dimension is an NP-hard task, it has been proved in [4]. It is known that determining a graph's partition dimension is an NP-hard problem as it is a generalization of finding the metric dimension. In different areas, both metric and partition

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dimensions contribute, strategies for mastermind games [5], such as network discovery [6], image processing, and game theory [3, 7].

Based on the characteristics of the partition dimension, it seems sensible to inquire about the graph characterizations. Researchers are constantly trying to establish if a family of a network's partition dimension is constant, finite, or unbounded. A number of conclusions are drawn from the study of a graph's partition dimension, particularly, diameter 2 and graph of order $n \geq 11$, then partition dimension is $n-3$ were discussed by Baskoro et al. [8]. As determined by Mehreen et al. [9], the fullerene graph's partition dimension is 3. For honeycomb and hexagonal networks, Rajan et al. [10] computed the partition dimension. The partition dimension of rooted product graphs was found by Monica and Santhakumar in [11]. The partition dimension of particular classes of series-parallel graphs has been investigated by Monica et al. [12]. For more literature on graph partition, refer [13–16].

The partition dimension of any general graph remains a challenging problem. In this paper, we examine the partition dimension for oxide networks, zigzag benzene networks, the subdivision of benzenoid hydrocarbon networks, and the subdivision of triangular benzene networks.

2 Preliminaries and basic concepts

Let $B = (V, E)$ be a simple and connected graph. The shortest possible route between two vertices v_1, v_2 is indicated by $d(v_1, v_2)$ and equals the distance between them. The distance between a vertex m and a set $S \subseteq V(B)$ is defined as $d(m, S) = min\{\ d(m, s),\quad s \in S\}$. Let $\pi = \{P_1, P_2, P_3, \ldots, P_k\}$ be a k partition of the vertex set $V(G)$ and $r(b\backslash\pi) = \{d(b, P_1), d(b, P_2), d(b, P_3), \ldots, d(b, P_k)\}$ be an k-tuple representation of a vertex m with regard to π. A resolving partition exists if and only if all representation codes of the vertex set of B are distinct with respect to π. The partition with the smallest cardinality of k is known as the partition dimension of B and is represented by the symbol $pd(B)$. In this work, we have identified certain chemical structures in which the partition dimension will be equal to 3.

The following theorems are highly useful for determining a connected graph's partition dimension of any graph B.

Theorem 2.1. *([4]). Suppose π is a resolving partition of $V(B)$, and let m and l be vertices in $V(B)$. If the distance from m to any vertex w in $V(B)$ (excluding m and l) is equal to the distance from l to the same vertex w, then m and l belong to different elements of π.*

Theorem 2.2. *([4]). Suppose that B is a simple, connected and undirected graph of order n. Then the following statements hold*

1. *$pd(B) = 2$ if and only if B is a path on n vertices.*
2. *$pd(B) = n$ if and only if B is a complete graph on n vertices.*

The rest of the article has been divided into the following sections. In Section 3 the partition dimension of oxide networks is calculated. In Section 4, the partition dimension of zigzag benzene networks is obtained. In Sections 5 and 6 the partition dimension of the subdivision of benzenoid hydrocarbon and triangular benzene networks are discussed. Applications and concluding remarks are provided in Sections 7 and 8, respectively.

3 Partition dimension of oxide networks

Oxide networks have a wide range of applications in chemistry and various fields due to their

diverse properties and structures. Some applications are electrochemical energy storage, photocatalysis, semiconductors, waste water treatment, nanomaterials, solar cells, and corrosion protection. The versatility of oxide networks in terms of structure, composition, and properties makes them indispensable in modern chemistry and materials science. Their applications continue to expand as new materials and technologies are developed.

In this section, we will determine the partition dimension of the oxide networks $OX(t)$, $t \geq 2$. The study of oxide networks has an importance for the study of silicate networks. A silicate network is transformed into an oxide network when all the silicon nodes are removed from it. In the oxide network, there are $9t^2 + 3t$ nodes and $18t^2$ edges. Although $OX(t)$ is a subgraph of $SL(t)$, and $OX(t)$ is more important for understanding $SL(t)$'s characteristics.

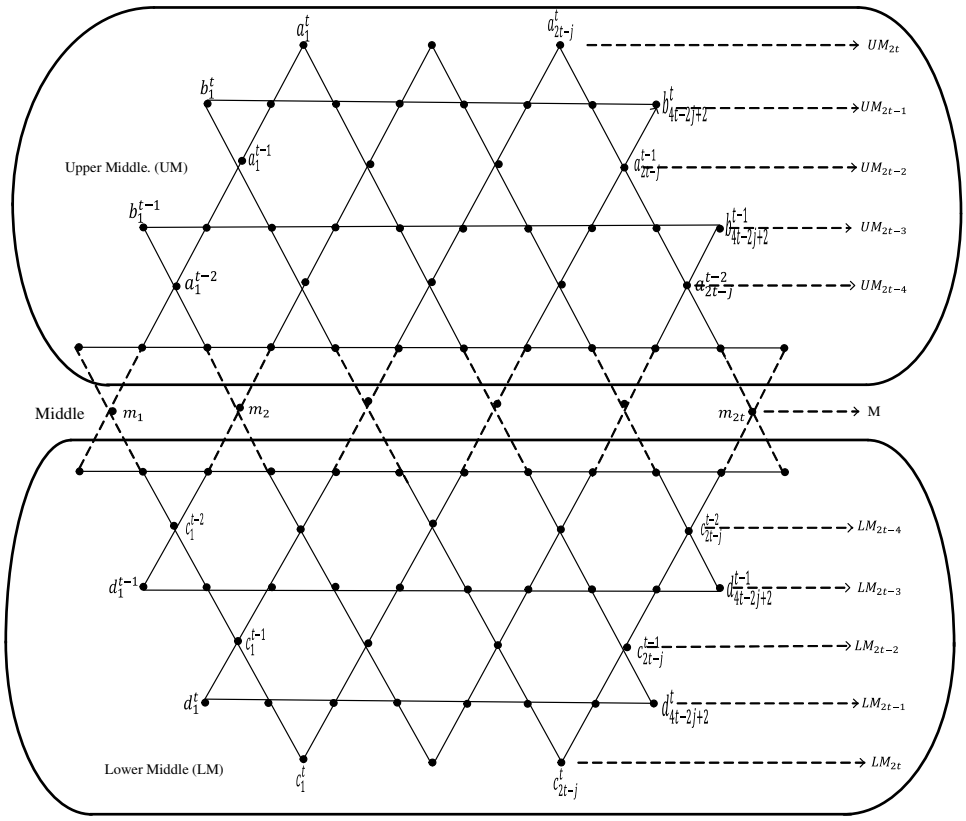


Figure 1: Labeling of oxide networks of dimension t.

Theorem 3.1. *Let $OX(t)$ be the t-dimensional oxide networks. Then $pd(OX(t)) = 3$, $t \geq 2$.*

Proof. An oxide network has a vertex set containing 3 regions, say middle (M), upper middle (UM), and the lower middle (LM). Label the middle region as $m_1, m_2, m_3, \ldots, m_{2t}$ from left to right, respectively. The upper middle has $2t$ levels, say $UM_1, UM_2, UM_3, \ldots, UM_{2t}$ starting next from the middle region. Label the vertices in UM_{2j} as $a_i^j, 1 \leq j \leq t, 1 \leq i \leq 2t - j$ from left to right, respectively and label the vertices in UM_{2j-1} as $b_i^j, 1 \leq j \leq t,\ 1 \leq i \leq 4t - 2j + 2$ from left to right, respectively. Similarly, the lower middle has $2t$ levels, say $LM_1, LM_2, LM_3, \ldots, LM_{2t}$ starting next from the middle region. Label the vertices in LM_{2j} as $c_i^j, 1 \leq j \leq t, 1 \leq i \leq 2t - j$ from left to right, respectively and label the vertices in LM_{2j-1}

as $d_i^j, 1 \leq j \leq t, 1 \leq i \leq 4t-2j+2$ from left to right, respectively. The labeling of the oxide network of dimension t is shown in Figure 1.

Let $\pi = \{S_1, S_2, S_3\}$, where $S_1 = \{b_1^i, 1 \leq i \leq t\}$, $S_2 = \{c_i^t, 1 \leq i \leq 2t-j\}$, and $S_3 = V(OX(t)) \setminus \{S_1 \cup S_2\}$, be a partition of $V(OX(t))$.

To prove π is a resolving partition of $OX(t)$, it is enough to prove that the representation of each vertex of $OX(t)$ is different with respect to π.

For $1 \leq i \leq 2t$ the representation of each vertex m_i of $OX(t)$ with respect to π is $r(m_i|\pi) = (2i-1, 2t, 0)$.

For $1 \leq j \leq t$ and $1 \leq i \leq 2t-j$ the representation of each vertex a_i^j of $OX(t)$ with respect to π is as follows:

$$r(a_i^j|\pi) = \begin{cases} (2i-1, 2t+2j, 0) & \text{if } 1 \leq j \leq t-1 \text{ and } 1 \leq i \leq 2t-j \\ (2i, 2t+2j, 0) & \text{if } 1 \leq i \leq 2t-j \text{ and } j = t. \end{cases}$$

For $1 \leq j \leq t$ and $2 \leq i \leq 4t-2j+2$, the representation of each vertex b_i^j of $OX(t)$ with respect to π is $r(b_i^j|\pi) = (i-1, 2t+2j-1, 0)$.

For $1 \leq j \leq t$, $2 \leq i \leq 4t-2j+2$, the representation each vertex c_i^j of $OX(t)$ with respect to π is $r(c_i^j|\pi) = (2i+2j-1, 2t-2j, 0)$.

For $1 \leq j \leq t$ and $1 \leq i \leq 4t-2j+1$, the representation of each vertex d_i^j of $OX(t)$ with respect to π is as follows.

$$r(d_i^j|\pi) = \begin{cases} (2j, 2t-2j+2, 0) & \text{if } i = 1 \text{ and } 1 \leq j \leq t \\ (i+2j-2, 2t-2j+1, 0) & \text{if } 2 \leq i \leq 4t-2j+1 \\ & \text{and } 1 \leq j \leq t \\ (i+2j-2, 2t+2-2j, 0) & \text{if } i = 4t-2j+2 \text{ and } 1 \leq j \leq t. \end{cases}$$

From the above representation of each vertex with respect to π we get $r(a|\pi) \neq r(b|\pi)$ for any $a, b \in V(OX(t))$. Hence $pd(OX(t)) = 3$. ■

4 Partition dimension of zigzag benzene networks

A zigzag benzenoid structure is a specific type of benzenoid hydrocarbon in graph theory. It consists of alternating single and double bonds in a zigzag pattern. It has more applications in various fields due to its unique structure and properties. In graph theory, you can represent this structure as a graph where atoms are represented by vertices, and bonds are represented by edges. The graph zigzag benzene structure is made up of p rows, each of which is made up of two hexagonal units that share one common edge. The first row of Z_t has eleven edges, while the second row has twenty-one edges. Following the same logic, we can conclude that Z_t has $10t+1$ edges and $8t+2$ vertices. Figure 2 displays the zigzag benzene structure of dimension 4.

Theorem 4.1. *Let Z_t be the zigzag benzene network of dimension t, $t \geq 2$, then $pd(Z_t) = 3$.*

Proof. The zigzag benzene networks has a vertex set containing 3 layers, say the Top layer X, the Middle layer Y, and the Bottom layer Z. Label the vertex in the top layer as $x_1, x_2, x_3, \ldots, x_{2t}$ from top to bottom, respectively. Label the vertex in the middle layer as $y_1, y_2, y_3, \ldots, y_{4t+2}$ from top to bottom, respectively. Label the vertices in the bottom layer as $z_1, z_2, z_3, \ldots, z_{2t}$ from top to bottom, respectively. The labeling of zigzag bebzene networks for dimension t is shown in Figure 3.

Let $\pi = \{P_1, P_2, P_3\}$ be a resolving partition of Z_t, where $P_1 = \{x_i, 1 \leq i \leq 2t\}$, $P_2 = \{y_{4t+2}\}$ and $P_3 = V(Z_t) \setminus \{P_1 \cup P_2\}$. To prove π is a resolving partition of Z_t, it is enough to

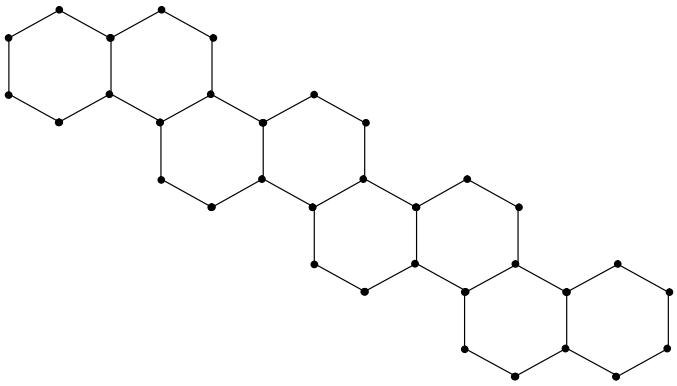


Figure 2: Zigzag benzene network of dimension 4.

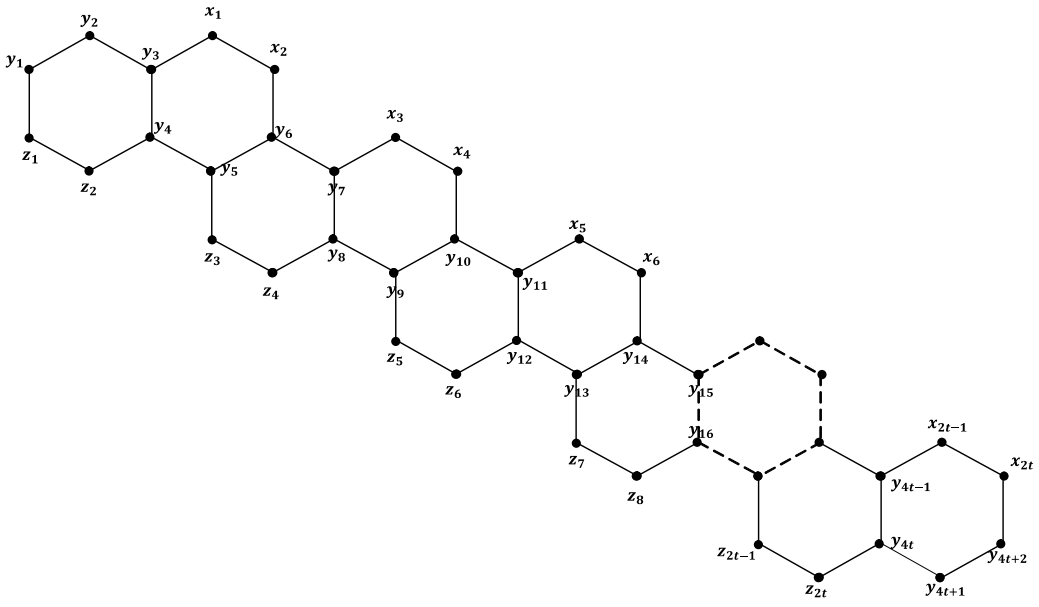


Figure 3: Labeling of zigzag benzene networks of dimension t.

prove all the vertices x_i, $1 \leq i \leq 2t$, y_i, $1 \leq i \leq 4t+2$ and z_i, $1 \leq i \leq 2t$ have distinct with respect to π.

For $1 \leq i \leq 2t$, the vertex representation of x_i in Z_t with respect to π is as follows.

$$r(x_i|\pi) = \begin{cases} (0, 4t-2i, 1) & \text{if } i = 1, 3, 5, \ldots, 2t-1 \\ (0, 4t-2i+1, 1) & \text{if } i = 2, 4, 6, \ldots, 2t. \end{cases}$$

For $1 \leq i \leq 4t+2$, the vertex representation of y_i in Z_t with respect to π is obtained as below:

$$r(y_i|\pi) = \begin{cases} (3, 4t+i, 0) & \text{if } i = 1 \\ (2, 4t+i-2, 0) & \text{if } i = 2 \\ (1, 4t+2-i, 0) & \text{if } i = 3, 6, 7, 10, 11, \ldots, 4t+2 \\ (2, 4t+2-i, 0) & \text{if } i = 4, 5, 8, 9, 12, 13 \ldots, 4t+1. \end{cases}$$

For $1 \leq i \leq 2t$, the vertex representation of z_i in Z_t with respect to π is as follows:

$$r(z_i|\pi) = \begin{cases} (4, 4t, 0) & \text{if } i = 1 \\ (3, 4t-2i+1, 0) & \text{if } i = 2, 4, 6, \ldots, 2t \\ (3, 4t-2i, 0) & \text{if } i = 3, 5, 7, \ldots, 2t-1. \end{cases}$$

From the above representation of each vertex with respect to π, we get $r(a|\pi) \neq r(b|\pi)$ for any $a, b \in V(Z_t)$. Hence $pd(Z_t) = 3$. ■

5 Partition dimension of subdivision of benzenoid hydrocarbon

Benzenoid hydrocarbons, which are a class of hydrocarbons characterized by their closed-ring structure similar to benzene, have several applications in various fields, such as dyes and pigments, pharmaceuticals, polycyclic aromatic hydrocarbons, material science, analytical chemistry, and fuel additives.

Polygons are utilized in various designs to construct honeycomb networks known as $HCN(t)$ [17], where t indicates the number of hexagons from the center to the boundary of the network. The outer border of $HCN(1)$ requires an additional layer of six hexagons in order to construct $HCN(2)$ from $HCN(1)$. $HCN(t)$ can be constructed by covering $HCN(t-1)$ with $6(t-1)$ hexagons. $SHCN(t)$ is derived from $HCN(t)$ by adding one vertex to each edge. Honeycomb networks have diverse applications in navigation, computer graphics, image processing, and cell phones. In chemistry, the honeycomb network is represented as benzenoid hydrocarbon [18]. Figure 4 displays examples of $HCN(2)$ and its one subdivision $SHCN(2)$. If $t \geq 2$, we have determined that the partition dimension of the honeycomb network with one subdivision is 3.

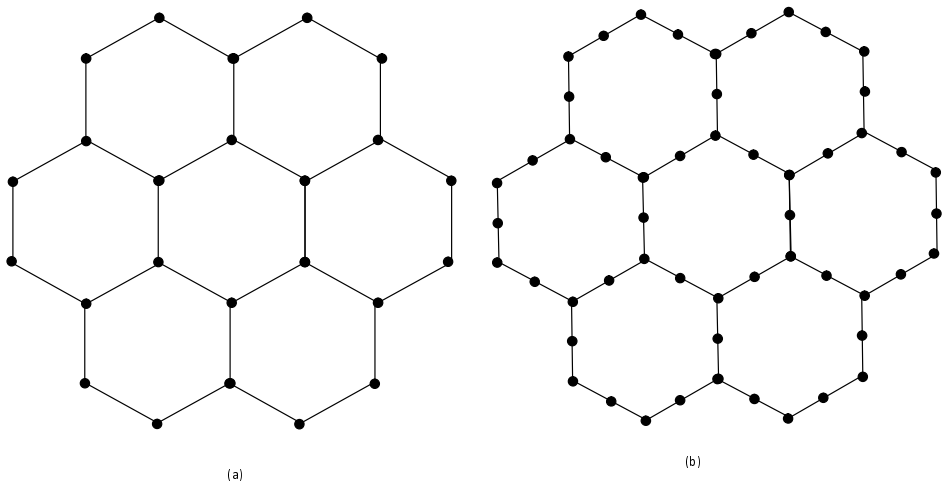


Figure 4: (a) Benzenoid hydrocarbon networks $HCN(2)$ (b) Subdivision of benzenoid hydrocarbon networks $SHCN(2)$.

Theorem 5.1. *Let B be a subdivision of benzenoid hydrocarbon networks of dimension t, $t \geq 2$, then $pd(B) = 3$.*

Proof. The one subdivision of the benzenoid hydrocarbon networks has a vertex set containing 3 regions, say middle (M), right middle (RM), and left middle (LM). Label the vertex in the middle region as $m_1, m_2, m_3, \ldots, m_{2t}$ from bottom to top, respectively. Label the vertices in RM_1 as $r_1, r_2, r_3, \ldots, r_{8t-3}$ from bottom to top, respectively. Label the vertices in LM_1 as $l_1, l_2, l_3, \ldots, l_{8t-3}$ from bottom to top, respectively. The left middle has $2t-1$ levels, say $LM_1, LM_2, \ldots, LM_{2t-1}$ starting next from the middle region. Similarly, the right middle has $2t-1$ levels, say $RM_1, RM_2, \ldots, RM_{2t-1}$ starting next from the middle region. Label the vertices in LM_{2j-1} as a_i^j, $2 \leq j \leq t-1$, $1 \leq i \leq 8t-4j-3$ from bottom to top, respectively. Label the vertices in LM_{2j} as b_i^j, $1 \leq j \leq t-1, 1 \leq i \leq 2t-j+1$ from bottom to top, respectively. Label the vertices in RM_{2j-1} as p_i^j, $2 \leq j \leq t-1$, $1 \leq i \leq 8t-4j-3$ from bottom to top, respectively. Label the vertices in RM_{2j} as q_i^j, $1 \leq j \leq t-1, 1 \leq i \leq 2t-j+1$ from bottom to top, respectively. The labeling of benzenoid hydrocarbon networks with one subdivision for dimension t is shown in Figure 5.

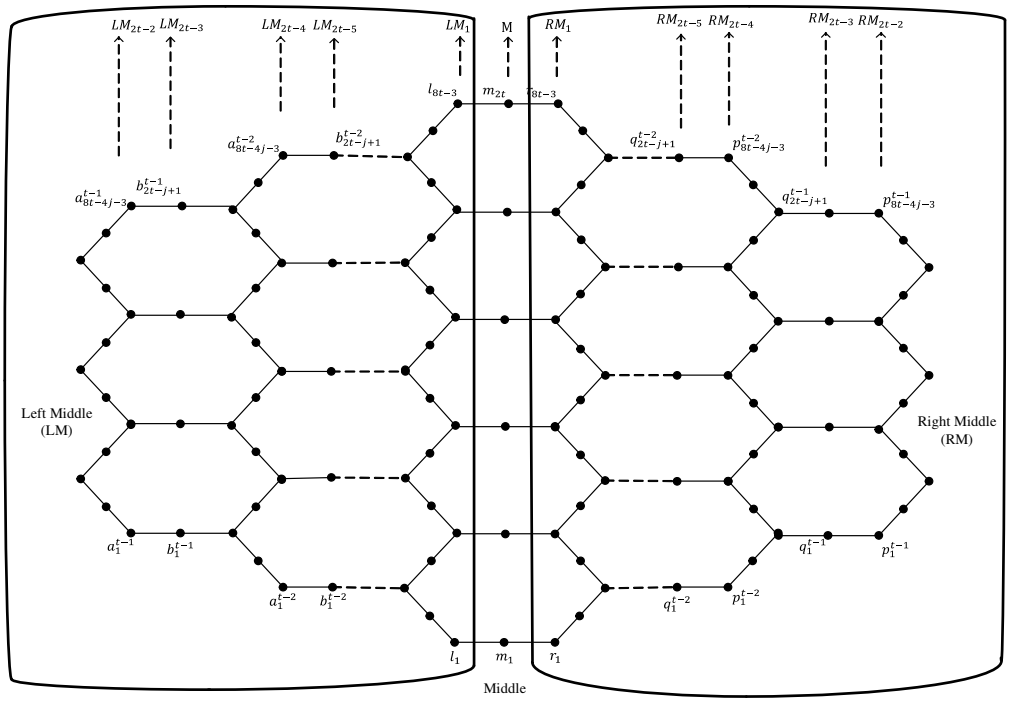


Figure 5: Labeling of Benzenoid hydrocarbon network of dimension t with one subdivision $SHCN(t)$.

Let $\pi = \{P_1, P_2, P_3\}$, where $P_1 = \{l_1\}$, $P_2 = \{a_i^{t-1};\ 1 \leq i \leq 8t-4j-3\}$ and $P_3 = V(B) \setminus \{P_1 \cup P_2\}$ be a resolving partition set of B. To prove π is a resolving partition it is enough to show that all the vertices in $V(B)$ has a unique representation with respect to π.

For $1 \leq i \leq 2t$, the vertex representation of m_i in $SHCN(t)$ with respect to π as $r(m_i|\pi) = (4i-3, 4t-3, 0)$.

For $2 \leq i \leq 8t-3$, the vertex representation of l_i in $SHCN(t)$ with respect to π is equal to:

$$r(l_i|\pi) = \begin{cases} (i-1, 4t-5, 0) & \text{if } i = 2, 4, 6, \ldots, 8t-3 \\ (i-1, 4t-6, 0) & \text{if } i = 3, 7, 11, \ldots, 8t-3 \\ (i-1, 4t-4, 0) & \text{if } i = 5, 9, 13, \ldots, 8t-3. \end{cases}$$

For $1 \leq i \leq 8t-3$, the vertex representation of r_i in $SHCN(t)$ with respect to π is obtained as below:

$$r(r_i|\pi) = \begin{cases} (i+1, 4t-1, 0) & \text{if } i = 2, 4, 6, \ldots, 8t-3 \\ (i+1, 4t, 0) & \text{if } i = 3, 7, 11, \ldots, 8t-3 \\ (i+1, 4t-2, 0) & \text{if } i = 1, 5, 9, 13, \ldots, 8t-3. \end{cases}$$

For $1 \leq j \leq t-2$, $1 \leq i \leq 8t-4j-3$, the vertex representation of a_i^j in $SHCN(t)$ with respect to π is as follows.

$$r(a_i^j|\pi) = \begin{cases} (4j+i-1, 4t-4j-5, 0) & \text{if } i = 2, 4, 6, \ldots, 8t-4j-3 \text{ and } 1 \leq j \leq t-2 \\ (4j+i-1, 4t-4j-4, 0) & \text{if } i = 5, 9, 13, \ldots, 8t-4j-3, \text{ and } 1 \leq j \leq t-2 \\ (4j+i-1, 4t-4j-6, 0) & \text{if } i = 3, 7, 11, \ldots, 8t-4j-3, \text{ and } 1 \leq j \leq t-2. \end{cases}$$

For $1 \leq j \leq t-1$, $1 \leq i \leq 2t-j+1$, the vertex representation of b_i^j of $SHCN(t)$ with respect to π is $r(b_i^j|\pi) = (4(j+i)-5, 4t-4j-3, 0)$.

For $1 \leq j \leq t-1$, $1 \leq i \leq 8t-4j-3$, the vertex representation of p_i^j of $SHCN(t)$ with respect to π is as follows:

$$r(p_i^j|\pi) = \begin{cases} (4j+i-1, 4t+4j-1, 0) & \text{if } i = 2, 4, \ldots, 8t-4j-3 \text{ and } 1 \leq j \leq t-1 \\ (4j+i-1, 4t+4j-2, 0)) & \text{if } i = 5, 9, \ldots, tn-4j-3, \text{ and } 1 \leq j \leq t-1 \\ (4j+i-1, 4t+4j, 0) & \text{if } i = 1, 3, \ldots, 8t-4j-3, \text{ and } 1 \leq j \leq t-1. \end{cases}$$

For $1 \leq j \leq t-1$, $1 \leq i \leq 2t-j+1$ the vertex representation of q_i^j in $SHCN(t)$ with respect to π is $r(q_i^j)|\pi) = (4(j+i)-3, 4t+4j-3, 0)$.

From the above representation of each vertex with respect to π we get $r(a|\pi) \neq r(b|\pi)$ for any $a, b \in V(B)$. Hence $pd(B) = 3$. ■

6 Partition dimension of subdivision of a triangular benzene networks

Triangular benzenoid networks' unique electronic properties make them suitable for optoelectronic applications, including light-emitting devices, photodetectors, and photovoltaic devices. Triangular benzenoid networks represent a fascinating area in chemical research, providing a wide array of potential applications across various scientific and technological fields. Their unique structure and properties open up possibilities for innovative advancements in materials science, nanotechnology, and numerous other domains within chemistry.

The triangular benzene network T_t is a generalization of the benzene molecule C_6H_6 in which the benzene rings create a triangle shape. The triangular benzenoid networks is constructed from up of hexagons organized in rows, with one hexagon increasing in each row. The benzene molecule is extremely important in the production of aromatic chemicals. Figure 6 illustrates the graphical structures of triangular benzenoid T_3 as well as the one subdivision of triangular benzenoid ST_3 of dimension 3.

Theorem 6.1. *Let B be a triangular benzene networks of dimension t with one subdivision, ST_t, $t \geq 2$, then $pd(B) = 3$.*

Proof. The strip between two successive lines is marked in one subdivision of triangular benzene networks are called the segments and it's denoted by S_i, $0 \leq i \leq 4t+2$. To find the minimum

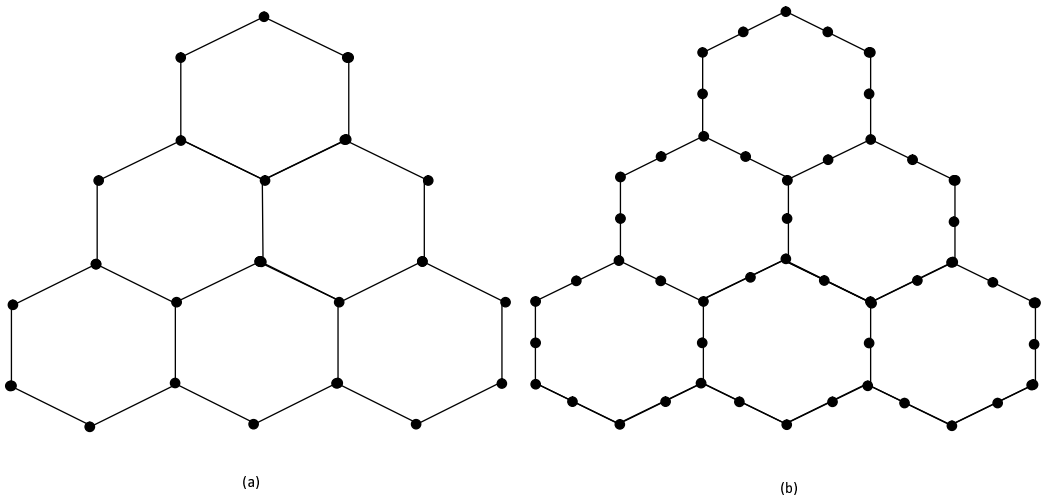


Figure 6: (a) Triangular benzene networks T_3 (b) subdivision of triangular benzene networks ST_3.

resolving partition vertex set of one subdivision of triangular benzene networks we need to label every vertex of the segments. Label the vertex of S_0 as $x_{0,1}$, label the vertices in S_1 as $x_{1,1}, x_{1,2}$ from left to right. Label the vertices in S_2 as $x_{2,1}, x_{2,2}$ from left to right. Continue the same logic and label the vertices in S_{4t+2} as $x_{4t+2,1}$, $x_{4t+2,2}$, $x_{4t+2,3}$, $\ldots$, $x_{4t+2,t}$ from left to right onwards. For illustration, the segment labeling of one subdivision of triangular benzene networks is shown in Figure 7.

Let $\pi = \{P_1, P_2, P_3\}$ be a resolving partition of ST_t, where $P_1 = \{x_{0,1}\}$, $P_2 = \{x_{i,1}, 1 \leq i \leq 4t+2\}$ and $P_3 = V(ST_t) \setminus \{P_1 \cup P_2\}$. To prove π is a resolving partition of ST_t, it is enough to prove that all the segments of the vertices $S_0, S_1, S_2, \ldots, S_{4t+2}$ have distinct with respect to π.

For $i = 0$, the vertex representation of S_i in ST_t with respect to π is as $r(S_i|\pi) = (0, i+1, i+1)$. For $i = 1$, the vertex representation of S_i in ST_t with respect to π is as $r(S_i|\pi) = (i, 2j-2, 4-2j), 1 \leq j \leq 2$.

For $i = 2$ the vertex representation of S_i in ST_t with respect to π is $r(S_i|\pi) = (i, 3j-3, 6-3j), 1 \leq j \leq 2$. For $i = 3$ the vertex representation of S_i in ST_t with respect to π is $r(S_i|\pi) = (i, 4j-4, 4-2j), 1 \leq j \leq 2$. For $i = 4$ the vertex representation of S_i in ST_t with respect to π is $r(S_i|\pi) = (i, 4j-4, 2-j)$ $1 \leq j \leq 2$.

For $i = 5, 9, 13, \ldots, 4t-3$ the vertex representation of S_i in ST_t with respect to π is as follows:

$$r(S_i|\pi) = \begin{cases} (i, j-1, j+1) & \text{if } j = 1 \\ (i, 2j-3, 0) & \text{if } 2 \leq j \leq \frac{i+3}{2}. \end{cases}$$

For $i = 6, 10, 14 \ldots, 4t-2$ the vertex representation of S_i in ST_t with respect to π is obtained as below:

$$r(S_i|\pi) = \begin{cases} (i, j-1, j+2) & \text{if } j = 1 \\ (i, 4j-6, 0) & \text{if } 2 \leq j \leq \frac{i+2}{4}. \end{cases}$$

For $i = 7, 11, 15 \ldots, 4t-1$ the vertex representation of S_i in ST_t with respect to π is as follows:

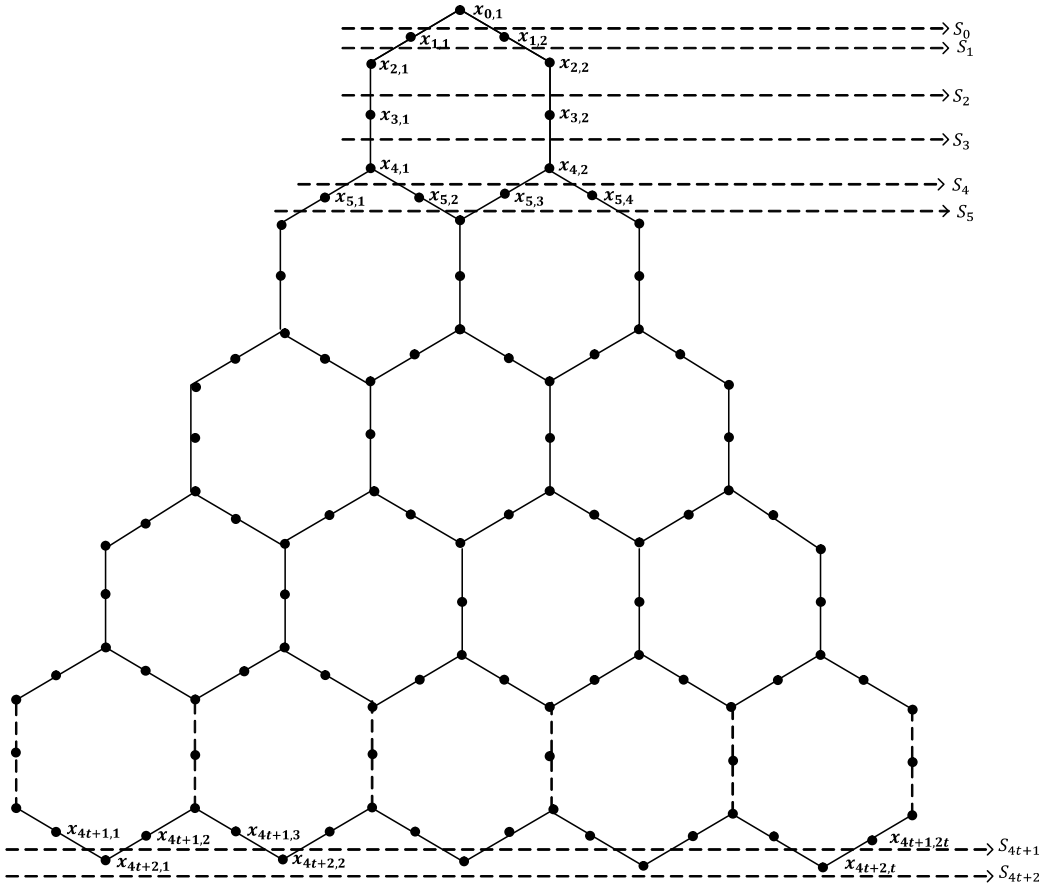


Figure 7: Segment labeling of one subdivision of triangular benzene networks of dimension t, ST_t.

$$r(S_i|\pi) = \begin{cases} (i, j-1, j+1) & \text{if } j = 1 \\ (i, 4j-5, 0) & \text{if } 2 \leq j \leq \frac{i+1}{4}. \end{cases}$$

For $i = 8, 12, 16 \ldots, 4t$ the vertex representation of S_i in ST_t with respect to π is equal to:

$$r(S_i|\pi) = \begin{cases} (i, j-1, j) & \text{if } j = 1 \\ (i, 4j-4, 0) & \text{if } 2 \leq j \leq \frac{i}{4}. \end{cases}$$

For $i = 4t+1$ the vertex representation of S_i in ST_t with respect to π is as follows:

$$r(S_i|\pi) = \begin{cases} (i, j-1, j+1) & \text{if } j = 1 \\ (i, 2j-3, 0) & \text{if } 2 \leq j \leq 2t-1. \end{cases}$$

For $i = 4t+2$ the vertex representation of S_i in ST_t with respect to π is obtained as below:

$$r(S_i|\pi) = \begin{cases} (i, j-1, j) & \text{if } j = 1 \\ (i, 4j-4, 0) & \text{if } 2 \leq j \leq t-1. \end{cases}$$

From the above representation of each vertex with respect to π we get $r(a|\pi) \neq r(b|\pi)$ for any $a, b \in V(ST_t)$. Hence $pd(ST_t) = 3$. ■

7 Applications of partition dimension in chemistry

The concept of partition dimension in graph theory has a wide range of applications beyond network design. Also, it can be applied in the domain of chemical graph theory and molecular modeling, which includes chemical structure analysis, drug discovery, chemoinformatics, molecular conformation analysis, structural isomerism, and chemical connectivity analysis. Additional information about each application can be found below.

Chemical structure analysis: Chemical compounds can be represented as molecular graphs, where atoms are vertices, and bonds are edges. Minimizing the partition dimension of these graphs can help in identifying unique structural features in chemical compounds, aiding in the analysis of chemical structures.

Drug discovery: In the field of pharmaceutical chemistry, partition dimension concepts can be used to analyze the structural diversity of chemical compounds, identify common structural motifs or substructures, and use this knowledge to aid in the design and discovery of new drugs. This approach can lead to more efficient drug development, the repurposing of existing drugs, and the identification of compounds with therapeutic potential.

Chemoinformatics: Chemoinformatics is an interdisciplinary field that combines concepts from chemistry, computer science, and information science to manage and analyze chemical information. In chemoinformatics, the partition dimension deals with the storage, retrieval, and analysis of chemical data. Partition dimension techniques can also be applied to enhance the efficiency of chemical database searching, substructure searching, data compression, and chemical information retrieval.

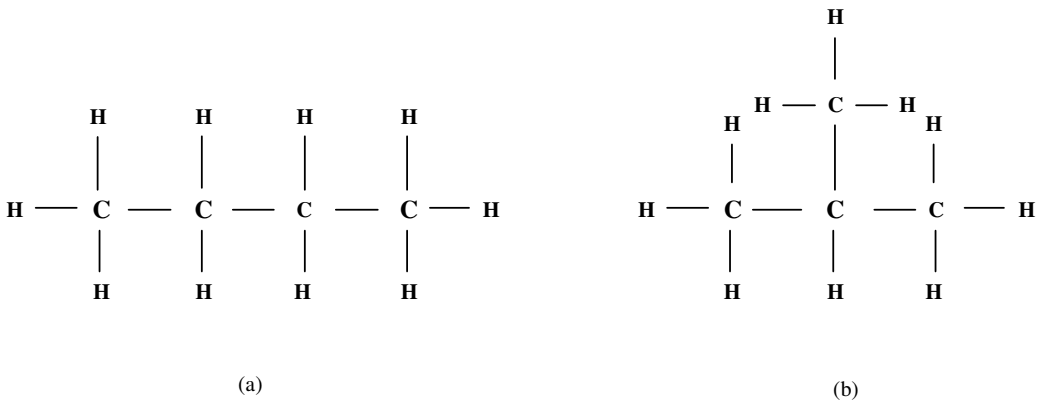
Molecular conformation analysis: Partition dimension methods can help analyze the possible conformations of a molecule. This is particularly important when studying the spatial arrangement of atoms in complex molecules.

Structural isomerism: Structural isomerism refers to a type of isomerism in chemistry where two or more compounds have the same molecular formula (i.e., they contain the same types and numbers of atoms) but differ in the connectivity or spatial arrangement of those atoms. In the context of structural isomerism, partition dimension can be a useful concept for distinguishing between these different isomers and identifying the unique structural features that set them apart.

Consider the isomers of C_4H_{10}, which have the molecular formula of butane. There are two main structural isomers: n-butane and isobutane see Figure 8. Partitioning these isomers could involve looking at the connectivity of carbon atoms and distinguishing between linear and branched arrangements.

For a more complex example, consider the structural isomers of C_6H_{12}, which have the molecular formula of hexane. Partitioning could involve identifying unique arrangements of carbon atoms in the chain, such as straight-chain hexane, branched-chain hexane, or cyclohexane.

Chemical connectivity analysis: Chemical connectivity analysis is vital for understanding the structure and reactivity of molecules. Minimizing the partition dimension simplifies the representation of molecular connectivity, making it easier to work with, store, retrieve, and analyze chemical data, which is essential in various fields of chemistry and related disciplines.



(a) (b)

Figure 8: Two main structural isomers of C_4H_{10}, (a) n-Butane (b) Iso-Butane.

8 Concluding remarks

This paper thoroughly investigated the structures of $OX(t)$, Z_t, $SHCN(t)$, and ST_t and established that the partition dimensions of all networks are 3 for $t \geq 2$. Further, the partition dimension of related interconnection networks is under investigation. It is important to note that the applications of the networks considered in this paper are still a subject of ongoing research and development, and their full potential is yet to be realized in many areas. As scientists continue to explore and understand their properties, more applications may emerge in the future.

Conflicts of Interest. The authors declare that they have no conflicts of interest regarding the publication of this article.

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References

[1] A. R. Meyer, Communication networks, In Mathematics for Computer Science, MIT Open Learning Library: Cambridge, MA, USA, 2010, Chapter 13, 253–272. https://people.csail.mit.edu/meyer/mcs.pdf.

[2] P. J. Slater, Leaves of trees, *Congr. Numer.* **14** (1975) 549–559.

[3] F. Harary and R. A. Melter, On the metric dimension of a graph, *Ars Comb.* **2** (1976) 191–195.

[4] G. Chartrand, E. Salehi and P. Zhang, The partition dimension of a graph, *Aequ. Math.* **59** (2000) 45–54. https://doi.org/10.1007/PL00000127.

[5] V. Chvátal, Mastermind, *Combinatorica* **3** (1983) 325–329, https://doi.org/10.1007/BF02579188.

[6] Z. Beerliova, F. Eberhard, T. Erlebach, A. Hall, M. Hoffmann, M. Mihal'ak and L. S. Ram, Network discovery and verification, *IEEE J. Sel. Areas Commun.* **24** (2006) 2168–2181, https://doi.org/10.1109/JSAC.2006.884015.

[7] R. A. Melter and I. Tomescu, Metric bases in digital geometry, *Comput. Graph. Image Process.* **25** (1984) 113–121, https://doi.org/10.1016/0734-189X(84)90051-3.

[8] E. T. Baskoro and D. O. Haryeni, All graphs of order $n \geq 11$ and diameter 2 with partition dimension $n - 3$, *Heliyon* **6** (2020) p. e03694, https://doi.org/10.1016/j.heliyon.2020.e03694.

[9] N. Mehreen, R. Farooq and S. Akhter, On partition dimension of fullerene graphs, *AIMS Math.* **3** (2018) 343–352, https://doi.org/10.3934/Math.2018.3.343.

[10] B. Rajan, A. William, I. Rajasingh, C. Grigorious and S. Stephen, On certain networks with partition dimension three, *In Proc. Int. Conf. Math., Eng. Bus. Manage.* (2012) 169–172.

[11] M. C. Monica and S. Santhakumar, Partition dimension of rooted product graphs, *Discrete Appl. Math.* **262** (2019) 138–147, https://doi.org/10.1016/j.dam.2019.02.007.

[12] C. M. Mohan, S. Santhakumar, M. Arockiaraj and J. B. Liu , Partition dimension of certain classes of series parallel graphs, *Theoret. Comput. Sci.* **778** (2019) 47–60, https://doi.org/10.1016/j.tcs.2019.01.026.

[13] Y. M. Chu, M. F. Nadeem, M. Azeem and M. K. Siddiqui, On sharp bounds on partition dimension of convex polytopes, *IEEE Access* **8** (2020) 224781–224790, https://doi.org/10.1109/ACCESS.2020.3044498.

[14] S. Pirzada and M. Aijaz, On graphs with same metric and upper dimension, *Discrete Math. Algorithms Appl.* **13** (2021) p. 2150015, https://doi.org/10.1142/S1793830921500154.

[15] S. Pirzada, M. Aijaz and S. P. Redmond, On upper dimension of graphs and their bases sets, *Discrete Math. Lett.* **3** (2020) 37–43.

[16] S. Pirzada, M. Aijaz and S. P. Redmond, Upper dimension and bases of zero-divisor graphs of commutative rings, *AKCE Int. J. Graphs Comb.* **17** (2020) 168–173, https://doi.org/10.1016/j.akcej.2018.12.001.

[17] P. Manuel, R. Bharati, I. Rajasingh and C. Monica, On minimum metric dimension of honeycomb networks, *J. Discrete Algorithms* **6** (2008) 20–27, https://doi.org/10.1016/j.jda.2006.09.002

[18] G. R. Roshini, S. B. Chandrakala, M. Vishu Kumar and B. Sooryanarayana, Non-neighbor topological indices of honeycomb networks, *Palest. J. Math.* **10** (2021) 52–58.