Iranian J. Math. Chem. 13 (4) December (2022) 291–299

IRANIAN JOURNAL OF
Mathematical Chemistry
Journal homepage: ijmc.kashanu.ac.ir

*Review Paper*

# On Color Matrix and Energy of Semigraphs

**ARDHENDU KUMAR NANDI[1], IVAN GUTMAN[2] AND SURAJIT KUMAR NATH[3,•]**

[1]Department of Mathematics, Basugaon College, P.O. Basugaon, 783372, India
[2] Faculty of Science, University of Kragujevac, P.O. Box 60, 34000 Kragujevac, Serbia
[3] Department of Mathematical Sciences, Bodoland University, P.O. Kokrajhar, 783370, India

**ARTICLE INFO**

**Article History:**
Received: 20 March 2022
Accepted: 14 June 2022
Published online: 30 December 2022
Academic Editor: Boris Furtula

**Keywords:**
Semigraph
Coloring of semigraph
Color matrix
Color energy

**ABSTRACT**

We introduce the concept of color matrix and color energy of semigraphs. The color energy is the sum of the absolute values of the eigenvalues of the color matrix. Some properties and bounds on color energy of semigraphs are established.



## 1. INTRODUCTION

Adiga *et al*. [2] introduced the concept of graph coloring, the color matrix, and its energy. Their definitions are as follows:

**Color matrix of a graph.** [2, 3] Let $G$ be a vertex-colored graph of order $n$. Then the *color matrix* of $G$ is the matrix $A_C(G) = (a_{ij})_{n\times n}$ for which

$$\begin{aligned} a_{ij} &= 1 \text{ if } v_i \text{ and } v_j \text{ are adjacent,} \\ &= -1 \text{ if } v_i \text{ and } v_j \text{ are non-adjacent with } c(v_i) = c(v_j), \end{aligned}$$

•Corresponding Author (Email address: surajitnathe9@gmail.com)
DOI: 10.22052/IJMC.2022.246186.1616

$= 0$, otherwise,

where $c(v_i)$ is the color of the vertex $v_i$ in $G$. Recall that, by definition [2], the vertices of the graph $G$ are colored so that two adjacent vertices always have different colors.

The color energy of a graph $G$ with respect to a given coloring is the sum of the absolute values of eigenvalues of the color matrix $A_C(G)$.

The first results on the color energy of graphs were communicated in [2]. Joshi and Joseph [4, 5] established some new bounds for the color energy. Motivated by the above mentioned works, we got interested to develop the concepts of color energy of semigraphs. Sampathkumar [6] in the year 1994 generalized the definition of graph to semigraph. Some definitions on semigraph are as follows:

**Semigraph.** A semigraph $G$ is an ordered pair $(V, E)$ where $V = \{v_1, v_2, \dots, v_n\}$ is a nonempty set whose elements called vertices of $G$ and $E = \{e_1, e_2, \dots, e_m\}$ is a set of $r$ −tuples, called edges of $G$. The edges consist of distinct vertices, for various $r \geq 2$, satisfying the following conditions:

i. Any two edges have at most one vertex in common.

ii. Two edges $(x_1, x_2, x_3, \dots, x_p)$ and $(y_1, y_2, y_3, \dots, y_q)$ are considered to be equal if and only if

(a) $p = q$ and

(b) either $x_i = y_i$ for $1 \leq i \leq p$, or $x_i = y_{p-i+1}$ for $1 \leq i \leq p$.

Thus the edge $e_i = (x_1, x_2, x_3, \dots, x_t)$ is same as the edge $(x_t, x_{t-1}, \dots, x_1)$ where $x_1$ and $x_t$ are said to be the end vertices, whereas $x_2, x_3, \dots, x_{t-1}$ are called the middle vertices of the edge $e_i$.

**Adjacent vertices.** Two vertices in a semigraph $G$ are said to be adjacent if they belong to the same edge and are said to be consecutively adjacent if in addition they are consecutive in order as well.

**Adjacent edges.** Two edges $e_i$ and $e_j$ in a semigraph $G$ are said to be adjacent if they have a common vertex.

**Degrees.** For a vertex $v$ in a semigraph $G$ we define various types of degrees as follows:

**i.** **Degree:** $\deg v$ is the number of edges having $v$ as an end vertex.

**ii.** **Edge degree:** $\deg_e v$ is the number of edges containing $v$.

**iii.** **Adjacent degree:** $\deg_a v$ is the number of vertices adjacent to $v$.

**iv.** **Consecutive adjacent degree:** $\deg_{ca} v$ is the number of vertices which are consecutively adjacent to $v$.

**Graphs associated with a given semigraph**. If $G = (V, E)$ be a semigraph of order $n$ and, size $m$, then three different graphs each having same vertex set $V$, pertain to the given semigraph $G$ as follows:

- *The end vertex graph* $G_e$: Two vertices in $G_e$ are adjacent if and only if they are end vertices of an edge in $G$.
- *The adjacency graph* $G_a$: Two vertices in $G_a$ are adjacent if and only if they are adjacent vertices in $G$.
- *The consecutive adjacency graph* $G_{ca}$: Two vertices in $G_{ca}$ are adjacent if and only if they are consecutive adjacent vertices in $G$.

**Vertex coloring of a semigraph.** A coloring of a semigraph $G = (V, E)$ is an assignment of colors to its vertices, such that not all vertices in an edge are equally colored. A strong coloring of $G$ is a coloring of vertices such that no two adjacent vertices are equally colored, whereas an $e$-coloring is a coloring of vertices such that no two adjacent end vertices of an edge are equally colored.

As $r$-coloring ($r$-strong coloring, $r$-$e$-coloring) uses $r$ colors, and partitions $V$ into $r$ respective color classes, each class consisting of vertices with the same color.

The chromatic number $\chi = \chi(G)$ of $G$ is the minimum number of colors needed in any coloring of $G$. Similarly we defined the strong chromatic number $\chi_s = \chi_s(G)$, and the $e$-chromatic number $\chi_e = \chi_e(G)$ of $G$. Clearly, a strong coloring is an $e$-coloring and an $e$-coloring is a coloring.

## 2. COLOR ENERGY OF SEMIGRAPHS

Let $G = (V, E)$ be a vertex-colored semigraph having $n$ vertices and $m$ edges. Some definitions relevant to its spectral properties are as follows:

**Definition 1. (**Color matrix of a semigraph) Let $G = (V, E)$ be a vertex-colored semigraph order $n$ and size $m$. Denote by $c(v_i)$ the color of the vertex $v_i$. Then the color matrix of the semigraph $A_C(G) = (a_{ij})_{n\times n}$ is defined as

$a_{ij}$ = 1 if $v_i$ and $v_j$ are adjacent.

= $-1$ if $v_i$ and $v_j$ are non-adjacent with $c(v_i) = c(v_j)$.

= 0, otherwise.

Recall that, in contrast to the case of vertex-colored graphs, adjacent vertices of a semigraph may have equal colors. On the other hand, it is clear that the color matrix of a semigraph $G$ is the color matrix of the adjacency graph $G_a$ associated with $G$.

**Definition 2.** (Color spectrum of a semigraph) If $A_C(G)$ be color matrix of a colored semigraph $G$. Then its eigenvalues $\xi_1, \xi_2, \xi_3, \dots, \xi_n$ are called color eigenvalues. The color matrix $A_C(G)$ is symmetric and hence all of color eigenvalues are real. If the distinct color eigenvalues of $A_C(G)$ are $\xi_1 > \xi_2 > \xi_3 > \ldots > \xi_r, r \le n$ with their multiplicities $m_1, m_2, \ldots, m_r$ then we have

$$Spec_c\, G = \begin{pmatrix} \xi_1 & \xi_2 & \cdots & \xi_r \\ m_1 & m_2 & \cdots & m_r \end{pmatrix},$$

called the color spectrum of a semigraph.

**Definition 3.** (Color energy of semigraph) The color energy of semigraph $G$ is defined as

$$\mathrm{E}_c(G) = \sum_{i=1}^{n} |\xi_i|.$$

This definition parallels the definition of the ordinary graph energy [1], and also of the color energy of a simple graph [2]. For a symmetric matrix, singular values are same as their eigenvalues. Therefore, the present definition of color energy $\mathrm{E}_c(G)$ of semigraph is consistent with the matrix energy of a semigraph [7], as well as with the definition of distance matrix and energy of a semigraph [8].

Suppose that $G = (V, E)$ is a semigraph of order *n*, and having *m* edges. Let $A_C(G)$ be the adjacency matrix with respect to a given coloring of $G = (V, E)$. Consider the characteristic polynomial of $A_C(G)$,

$$P_C(G, \xi) = \det(\xi I - A_C(G)) = a_0\xi^n + a_1\xi^{n-1} + a_2\xi^{n-2} + a_3\xi^{n-3} + \cdots + a_n.$$

**Lemma 1. [11]** If $A$ is a real or complex square matrix of order $n$ with eigenvalues $\xi_1, \xi_2, \xi_3, \ldots, \xi_n$, then for each $k \in \{1, 2, 3, \ldots, n\}$, the number $S_k = (-1)^k a_k$ = the sum of the $k \times k$ principal minors of $A$, where $a_k$'s are the coefficients of the characteristic polynomial of $A$ and $S_k$ the $k^{\text{th}}$ symmetric function of $\xi_1, \xi_2, \xi_3, \ldots, \xi_n$, is the sum of the products of the eigenvalues taken *k* at a time.

**Theorem 1.** Using the notations given above, we have

(a) $a_0 = 1$.

(b) $a_1 = 0$.

(c) $a_2 = -\sum_{i=1}^{m} {}^{|e_i|}C_2$- Number of pairs of non-adjacent vertices receiving the same color in $G$.

(d) $a_3 = -2$(Number of triangles of $G$ + No of triplet of which two adjacent vertices with same color -No of triplet of which two non adjacent vertices with same color - Number of non-adjacent triplet having same color in $G$).

**Proof.** (a) It is clear from the definition of the characteristic polynomial of $A_C(G)$. i.e. $P_C(G, \xi) = \det(\xi I - A_C(G))$, that $a_0 = 1$.

(b) Since the diagonal elements of $A_C(G)$ are all zeros, $a_1 = 0$.
(c) $(-1)^2 a_2 =$Sum of all the $2 \times 2$ principal minors of

$$A_C(G) = \sum_{1 \le i < j \le n} \begin{vmatrix} a_{ii} & a_{ij} \\ a_{ji} & a_{jj} \end{vmatrix} = \sum_{1 \le i < j \le n} \left( a_{ii} a_{jj} - a_{ij} a_{ji} \right) = -\sum_{1 \le i < j \le n} a^2{}_{ij}.$$

$a_2 = -\sum_{i=1}^{m} {}^{|e_i|}C_2$- Number of pairs of non-adjacent vertices receiving the same color in $G$.
(d) $a_3 = (-1)^3$Sum of all the $3 \times 3$ principal minors of

$$A_C(G) = (-1)^3 \sum_{1 \le i < j < k \le n} \begin{vmatrix} a_{ii} & a_{ij} & a_{ik} \\ a_{ji} & a_{jj} & a_{jk} \\ a_{ki} & a_{kj} & a_{kk} \end{vmatrix} = -2 \sum a_{ij} a_{jk} a_{ki}$$

$= -2$(Number of triangles of $G$ + No of triplet of which two adjacent vertices with same color - No of triplet of which two non adjacent vertices with same color - Number of non-adjacent triplet having same color in $G$). ■

**Lemma 2.** If $\xi_1, \xi_2, \xi_3, \ldots, \xi_n$ are the eigenvalues of the color matrix $A_C(G)$ of a semigraph $G(V, E)$ of order *n*, having $m$ edges, then $\sum_{i=1}^{n} \xi_i^2 = 2\left[\sum_{i=1}^{m} {}^{|e_i|}C_2 + m'_c\right]$ where $m'_c$ is the number of pairs of non-adjacent vertices receiving the same color and $|e_i|$is the number of vertices in the edge $e_i \in E$.

**Proof.** Consider

$$\sum_{i=1}^{n} \xi_i^2 = \sum_{i=1}^{n} \left( A_c{}^2 \right)_{ii} = \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} a_{ji}$$

$$= \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij}{}^2 \text{(As } A_C(G) \text{ is a symmetric matrix)}$$

$$= 2 \sum_{i<j} \left( a_{ij} \right)^2 + \sum_{i=1}^{n} (a_{ii})^2$$

$$= 2\left[\sum_{i-1}^{m} {}^{|e_i|}C_2 + m'_c\right] \text{(Since} \sum_{i=1}^{n} (a_{ii})^2 = 0\text{)}.$$ ■

**Lemma 3.** Let $G = (V, E)$ be a colored semigraph having *n* vertices and *m* edges. If $|e_i|$ is the number of vertices in the edge $e_i \in E$, then

$$\left( \sum_{i=1}^{m} {}^{|e_i|}C_2 + m'_c \right) \ge m.$$

Equality holds when $G$ is a graph.

**Proof.** Clearly, for a connected semigraph, $|e_i| \ge 2$ Thus, ${}^{|e_i|}C_2 \ge 1$, i.e. $\sum_{i=1}^{m} {}^{|e_i|}C_2 \ge m$. Hence $\sum_{i=1}^{m} {}^{|e_i|}C_2 + m'_c \ge m$. ■

**Lemma 4.** Let $G = (V, E)$ be a connected semigraph having $n$ vertices and $m$ edges. If $|e_i|$ is the number of vertices in the edge $e_i \in E$, then $n \leq 2\sum_{i=1}^{m} {}^{|e_i|}C_2$.

**Proof.** Clearly $n \leq \sum_{i=1}^{n} \deg_e v_i = \sum_{i=1}^{m} |e_i| \leq 2\sum_{i=1}^{m} {}^{|e_i|}C_2$. ■

**Theorem 2.** If the energy of a colored semigraph is a rational number, then it must be an even positive integer.

**Proof.** From the [12, Theorem 2.12], we have if $\xi_1, \xi_2, \xi_3, \ldots, \xi_n$ are color eigenvalues of $A_C(G)$, the adjacency matrix of a semigraph $G = (V, E)$ of order $n$ then, Trace of $|A_C(G)| = 0 = \sum_{i=1}^{n} \xi_i$. Of these eigenvalues, $\xi_1, \xi_2, \xi_3, \ldots, \xi_r$ are positive and the rest non-positive. Thus we have

$$\begin{aligned} E_C(G) &= \sum_{i=1}^{n} |\xi_i| \\ &= (\xi_1 + \xi_2, + \cdots + \xi_r) - (\xi_{r+1} + \xi_{r+2} + \cdots + \xi_n) \\ &= 2(\xi_1 + \xi_2, + \cdots + \xi_r). \end{aligned}$$

The sum $\xi_1 + \xi_2, + \cdots + \xi_r$ is an algebraic integer as $\xi_1, \xi_2, \xi_3, \ldots, \xi_r$ are algebraic integers. Hence $2(\xi_1 + \xi_2, + \cdots + \xi_r)$ must be an even positive integer if $E_C(G)$is rational.

■

## 3. SOME BOUNDS FOR COLOR ENERGY OF A SEMIGRAPH

**Theorem 3.** Let $G = (V, E)$ be a colored semigraph having $n$ vertices and $m$ edges. Then

$$E_c(G) \leq \sqrt{2n\left(\sum_{i=1}^{m} {}^{|e_i|}C_2 + m'_c\right)},$$

where $m'_c$ is the number of pairs of non-adjacent vertices in $G$ receiving the same color.

**Proof.** The color matrix of a semigraph $A_C(G)$ is symmetric and hence its color eigenvalues are real and can be ordered as $\xi_1 \geq \xi_2 \geq \xi_3 \geq \cdots \geq \xi_n$. Appling the Cauchy-Schwarz inequality, we have $(\sum_{i=1}^{n} u_i v_i)^2 \leq (\sum_{i=1}^{n} u_i)^2 (\sum_{i=1}^{n} v_i)^2$. Substituting $u_i = 1$, $v_i = |\xi_i|$ in the above inequality and by Lemma 2, we have

$$[E_c(G)]^2 = \left(\sum_{i=1}^{n} |\xi_i|\right)^2 \leq n\left(\sum_{i=1}^{n} |\xi_i|^2\right) = n\sum_{i=1}^{n} \xi_i^2 = 2n\left(\sum_{i=1}^{m} {}^{|e_i|}C_2 + m'_c\right).$$

Hence, $E_c(G) \leq \sqrt{2n\left(\sum_{i=1}^{m} {}^{|e_i|}C_2 + m'_c\right)}$. ■

**Theorem 4.** Let $G = (V, E)$ be a colored semigraph having $n$ vertices and $m$ edges, and let $m'_c$ be the number of pairs of non-adjacent vertices receiving the same color. Then

$$E_c(G) \geq \sqrt{2\left(\sum_{i=1}^{m} {}^{|e_i|}C_2 + m'_c\right) + n(n-1)\Delta^{2/n}},$$

where $\Delta = |\det A_c(G)|$.

**Proof.** In view of Definition 3 and Lemma 2 we have,

$$[E_c(G)]^2 = \left(\sum_{i=1}^{n} |\xi_i|\right)^2 = \sum_{i=1}^{n} \xi_i^2 + \sum_{i \neq j} |\xi_i|\,|\xi_j|.$$

By applying $AM \geq GM$, we have

$$\frac{1}{n(n-1)} \sum_{i\neq j} |\xi_i|\,|\xi_j| \geq \left(\prod_{i \neq j} |\xi_i||\xi_j|\right)^{1/n(n-1)} = \left(\prod_{i\neq j} |\xi_i|^{2(n-1)}\right)^{1/n(n-1)}$$

$$= \left(\prod_{i \neq j} |\xi_i|\right)^{2/n}$$

$$= \Delta^{2/n},$$

i.e. $\sum_{i \neq j} |\xi_i||\xi_i| \geq n(n-1)\Delta^{2/n}$.

Thus, $[E_c(G)]^2 \geq \sum_{i=1}^{n} {\xi_i}^2 + n(n-1)\Delta^{2/n} = 2\left(\sum_{i=1}^{m} {}^{|e_i|}C_2 + m'_c\right) + n(n-1)\Delta^{2/n}$.

Therefore $E_c(G) \geq \sqrt{2\left(\sum_{i=1}^{m} {}^{|e_i|}C_2 + m'_c\right) + n(n-1)\Delta^{2/n}}$. ■

**Theorem 5.** Let $G = (V, E)$ be a colored semigraph of order $n$ and size $m$. Let the color eigenvalues of $A_C(G)$ be $\xi_1 \geq \xi_2 \geq \xi_3 \geq \cdots \geq \xi_n$. Then

$$E_c(G) \leq |\xi_1| + \sqrt{(n-1)\left[2\left(\sum_{i=1}^{m} {}^{|e_i|}C_2 + m'_c\right) - \xi_1^2\right]},$$

where $m'_c$ is the number of pairs of non-adjacent vertices in $G$ receiving the same color.

**Proof.** Let $\xi_1 \geq \xi_2 \geq \xi_3 \geq \cdots \geq \xi_n$ be the color eigenvalues of $A_C(G)$. Appling the Cauchy-Schwarz inequality on to vectors $(|\xi_2|, |\xi_3|, \ldots, |\xi_n|)$ and $(1, 1, \ldots, 1)$ with $n-1$ entries,

$$\left(\sum_{i=2}^{n} |\xi_i|\right)^2 \leq (n-1)\left(\sum_{i=2}^{n} |\xi_i|^2\right),$$

i.e. $\sum_{i=2}^{n} |\xi_i| \leq \sqrt{(n-1)\left(\sum_{i=2}^{n} |\xi_i|^2\right)}$,

i.e. $\sum_{i=1}^{n} |\xi_i| - |\xi_1| \leq \sqrt{(n-1)\left(\sum_{i=2}^{n} |\xi_i|^2\right)}$.

By Definition 3 and Lemma 2, we have

$$E_c(G) \leq |\xi_1| + \sqrt{(n-1)\left[2\left(\sum_{i=1}^{m} {}^{|e_i|}C_2 + m'_c\right) - \xi_1^2\right]}.$$ ■

**Theorem 6.** Let $G = (V, E)$ be a colored semigraph of order $n$ and size $m$. Let $\xi_{\max}$ be the largest absolute value of a color eigenvalue. Then

$$E_c(G) \geq \frac{2\left[\sum_{i=1}^{m} {}^{|e_i|}C_2 + m'_c\right]}{\xi_{\max}},$$

where $m'_c$ is the number of pairs of non-adjacent vertices in $G$ receiving the same color.

**Proof.** Let $\xi_{\max}$ be the largest absolute value of the color eigenvalue of $A_C(G)$. Then $\xi_{\max}|\xi_i| \geq \xi_i^2$. Thus $\sum_{i=1}^{n} \xi_{\max}|\xi_i| \geq \sum_{i=1}^{n} \xi_i^2$, i.e. $\xi_{\max} \sum_{i=1}^{n}|\xi_i| \geq 2\left[\sum_{i=1}^{m} {}^{|e_i|}C_2 + m'_c\right]$, by Lemma 2. Hence

$$E_c(G) \geq \frac{2\left[\sum_{i=1}^{m} {}^{|e_i|}C_2 + m'_c\right]}{\xi_{\max}}.$$ ■

**Theorem 7.** Let $G = (V, E)$ be a colored semigraph of order $n$, size $m$, and $m'_c$ be the number of pairs of non-adjacent vertices in $G$ receiving the same color. Then

$$2\sqrt{\left(\sum_{i=1}^{m} {}^{|e_i|}C_2 + m'_c\right)} \leq E_c(G) \leq 2\left(\sum_{i=1}^{m} {}^{|e_i|}C_2 + m'_c\right).$$

**Proof.** Consider

$$[E_c(G)]^2 = \left(\sum_{i=1}^{n}|\xi_i|\right)^2 = \sum_{i=1}^{n}|\xi_i|^2 + \sum_{i \neq j}|\xi_i|\left|\xi_j\right| = \sum_{i=1}^{n}|\xi_i|^2 + 2\sum_{i<j}|\xi_i||\xi_j|. \quad (1)$$

By Lemma 1, we have

$$a_2 = (-1)^2 \times \text{Sum of all the } 2 \times 2 \text{ principal minors of } A_C(G) = \sum_{1 \leq i<j \leq n} \xi_i \xi_j.$$

Therefore, $\sum_{1 \leq i<j \leq n} \xi_i \xi_i = \sum_{1 \leq i<j \leq n} \begin{vmatrix} a_{ii} & a_{ij} \\ a_{ji} & a_{jj} \end{vmatrix} = \sum_{1 \leq i<j \leq n}(a_{ii} \; a_{jj} - a_{ij} \; a_{ji}),$

As color matrix $A_C(G)$ is symmetric, $a_{ij} = a_{ji}$ and $a_{ii} = 0, \forall i$. Thus

$$\sum_{1 \leq i<j \leq n} \xi_i \xi_j = -\sum_{1 \leq i<j \leq n} a_{ij} a_{ji} = -\sum_{1 \leq i<j \leq n}\left(a_{ij}\right)^2 = -\left[\sum_{i=1}^{m} {}^{|e_i|}C_2 + m'_c\right].$$

We know that, $\sum_{i<j} |\xi_i||\xi_j| \geq |\sum_{i<j} \xi_i \xi_j|$. Thus,

$$\sum_{i<j} |\xi_i||\xi_j| \geq \left|\sum_{i=1}^{m} {}^{|e_i|}C_2 + m'_c\right|. \quad (2)$$

Using equations (1) and (2) along with Lemma 2, we get

$$[E_c(G)]^2 \geq 4\left|\sum_{i=1}^{m} {}^{|e_i|}C_2 + m'_c\right|.$$

Taking positive square-root, we get

$$E_c(G) \geq 2\sqrt{\left|\sum_{i=1}^{m} {}^{|e_i|}C_2 + m'_c\right|}. \quad (3)$$

By Lemma 4 we have, $n \leq 2\sum_{i=1}^{m} {}^{|e_i|}C_2 \leq 2\left[\sum_{i=1}^{m} {}^{|e_i|}C_2 + m'_c\right]$. Thus,

$$2n\left[\sum_{i=1}^{m} {}^{|e_i|}C_2 + m'_c\right] \geq 4\left[\sum_{i=1}^{m} {}^{|e_i|}C_2 + m'_c\right]^2.$$

Taking positive square-root, we get

$$\sqrt{2n\left[\sum_{i=1}^{m} {}^{|e_i|}C_2 + m'\right]} \leq 2\left[\sum_{i=1}^{m} {}^{|e_i|}C_2 + m'_c\right].$$

Thus by using Theorem 3,

$$E_c(G) \geq 2\left[\sum_{i=1}^{m} {}^{|e_i|}C_2 + m'_c\right]. \tag{4}$$

Hence, from (3) and (4) we have

$$2\sqrt{\left|\sum_{i=1}^{m} {}^{|e_i|}C_2 + m'_c\right|} \leq E_c(G) \leq 2\left[\sum_{i=1}^{m} {}^{|e_i|}C_2 + m'_c\right]. \qquad \blacksquare$$

## REFERENCES

1. I. Gutman, The energy of a graph, *Ber. Math. Statist. Sekt. Forsch. Graz.* **103** (1978) 1–22.
2. C. Adiga, E. Sampathkumar, M. A. Sriraj and A. S. Shrikanth, Color energy of a graph, *Proc. Jangjeon Math. Soc.* **16** (3) (2013) 335–351.
3. E. Sampathkumar and M. A. Sriraj, Vertex labeled/colored graphs, matrices and signed graphs, *J. Comb. Inf. Syst. Sci.* **38** (2013) 113–120.
4. P. B. Joshi and M. Joseph, Further results on color energy of graphs, *Acta Univ. Sapientiae Inform.* **9** (2) (2017) 119–131.
5. P. B. Joshi and M. Joseph, On new bounds for color energy of graphs, *Int. J. Pure Appl. Math.* **117** (11) (2017) 25–33.
6. E. Sampatkumar, *Semigraphs and Their Applications*, 1st ed. Academy of Discrete Mathematics and Application, India, 2019.
7. Y. S. Gaidhani, C. M. Deshpande and S. Pirzada, Energy of a semigraph, *AKCE Int. J. Graphs Comb.* **16** (1) (2019) 41–49.
8. S. K. Nath, I. Gutman and A. Kumar Nandi, Distance Matrix and Energy of Semigraph, *Bull. Int. Math. Virtual Inst.* **11**(3) (2021) 597–603.
9. I. Gutman and O. E. Polansky, *Mathematical Concepts in Organic Chemistry*, Springer, Berlin, 1986.
10. S. K. Nath and A. Kumar Nandi, Adjacency matrix of signed semigraph, *Adv. Appl. Discrete Math.* **27** (2) (2021) 193–207.
11. R. Rehman and I. C. F. Ipsen, Computing characteristic polynomials from eigenvalues, *SIAM J. Matrix Anal. Appl.* **32** (1) (2011) 90–114.
12. C. Adiga, R Balakrishnan and W. So, The skew energy of a digraph, *Linear Algebra Appl.* **432** (7) (2010) 1825–1835.