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# Chemical Hyperstructures for Elements with Four Oxidation States

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**ABSTRACT**


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Algebraic hyperstructures represent a natural extension of classical algebraic structures and they have many applications in various sciences. In this paper, we investigate mathematical structures of chemical reactions for arbitrary elements with four oxidation states.

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## 1. INTRODUCTION

Algebraic hyperstructures represent a natural generalization of classical algebraic structures and they were introduced by Marty [20] in 1934 at the eighth Congress of Scandinavian Mathematicians, where he generalized the notion of a group to that of a hypergroup. A hypergroup is a non-empty set equipped with an associative hyperoperation and reproductive hyperoperation. In a group, the composition of two elements is an element whereas in a hypergroup, the composition of two elements is a non-empty set. A generalization of algebraic hyperstructures was introduced in 1990 by T. Vougiouklis [23] where he defined weak hyperstructures. Many researchers such as Corsini [7], Corsini and

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Leoreanu [8], Davvaz [9, 10], Davvaz and Leoreanu-Fotea [18], and Vougiouklis [22] wrote books related to hyperstructure theory and their applications. One of the motivations for the study of hyperstructures comes from chemical reactions. In [12], Davvaz and Dehghan-Nezhad provided examples of hyperstructures associated with chain reactions. In [14], Davvaz et al. introduced examples of weak hyperstructures associated with dismutation reactions. In [17], Davvaz et al. investigated the examples of hyperstructures and weak hyperstructures associated with redox reactions. Also, see [1, 3, 5, 6, 11, 15, 16]. In [1], the authors presented three different examples of weak hyperstructures associated to electrochemical cells. In [6], Chung et al. investigated mathematical structures of chemical reactions for three consecutive oxidation states of elements. Some authors considered some elements with four oxidation states and investigated their chemical hyperstructures. For example, Chun in [4] presented chemical hyperstructures of chemical reactions for a set of Titanium and the authors in [2] studied chemical hyperstructures of chemical reactions for a set of Astatine, a set of Tellurium and a set of Bismuth.

In this paper, we generalize the work done by the authors in [2] and by Chun in [4] by considering an arbitrary element with four oxidation states and investigating its algebraic hyperstructures and it is organized as follows: After an Introduction, Section 2 presents some definitions and concepts related to (weak) hyperstructures that are used throughout the paper. Section 3 presents chemical hyperstructures using redox reactions of an arbitrary element with four oxidation states as algebraic hyperstructures under certain conditions. Section 4 presents some examples of elements with four oxidation states that satisfy the conditions presented in Section 3.

## 2. PRELIMINARIES

In this section, we present some definitions and concepts related to (weak) hyperstructures that are used throughout the paper.

**Definition 2.1.** [9] Let  $H$  be a non-empty set. Then, a mapping  $\circ : H \times H \rightarrow P^*(H)$  is called a binary hyperoperation on  $H$ , where  $P^*(H)$  is the family of all non-empty subsets of  $H$ . The couple  $(H, \circ)$  is called a hypergroupoid.

In the above definition, if  $A$  and  $B$  are two non-empty subsets of  $H$  and  $x \in H$ , then we define  $A \circ B = \bigcup_{a \in A, b \in B} a \circ b$ ,  $x \circ A = \{x\} \circ A$  and  $A \circ x = A \circ \{x\}$ .  $H_v$ -structures were introduced by T. Vougiouklis as a generalization of the well-known algebraic hyperstructures [21, 23], also see [13, 22]. Some axioms of classical algebraic hyperstructures are replaced by their corresponding weak axioms in  $H_v$ -structures. Most of  $H_v$ -structures are used in representation theory.

**Definition 2.2.** [22] A hypergroupoid  $(H, \circ)$  is called an  $H_v$ -semigroup if for all  $x, y, z \in H$ ,  $(x \circ (y \circ z)) \cap ((x \circ y) \circ z) \neq \emptyset$ .

A subset  $K$  of an  $H_v$ -semigroup is an  $H_v$ -subsemigroup if  $K$  is an  $H_v$ -semigroup. An element  $x \in H$  is called idempotent if  $x^2 = x \circ x = x$  and an element  $e \in H$  is called an identity of  $(H, \circ)$  if  $x \in x \circ e \cap e \circ x$ , for all  $x \in H$ .

We present an example on a finite  $H_v$ -semigroup.

**Example 1.** [2] Let  $G = \{a, b, c, d\}$ ,  $\oplus$  be the hyperoperation on  $G$  and consider Table 1 corresponding to  $(G, \oplus)$ :

**Table 1:**  $(G, \oplus)$ .

$\oplus$	$a$	$b$	$c$	$d$
$a$	$a$	$\{a, c\}$	$\{a, c\}$	$c$
$b$	$\{a, c\}$	$\{a, c\}$	$\{a, c\}$	$c$
$c$	$\{a, c\}$	$\{a, c\}$	$c$	$\{c, d\}$
$d$	$c$	$c$	$\{c, d\}$	$d$

Then  $(G, \oplus)$  is a commutative  $H_v$ -semigroup.

**Definition 2.3.** [7] A hypergroupoid  $(H, \circ)$  is called a:

1. semihypergroup if for every  $x, y, z \in H$ , we have  $x \circ (y \circ z) = (x \circ y) \circ z$ .
2. quasihypergroup if for every  $x \in H$ ,  $x \circ H = H = H \circ x$ . The latter condition is called the reproduction axiom.
3. hypergroup if it is a semihypergroup and a quasi-hypergroup.

We present an example of a commutative hypergroup of three elements.

**Example 2.** [2] Let  $G = \{a, b, c\}$ ,  $\circ$  be the hyperoperation on  $G$  and consider Table 2 corresponding to  $(G, \circ)$ :

**Table 2:**  $(G, \circ)$ .

$\circ$	$a$	$b$	$c$
$a$	$a$	$\{a, b\}$	$G$
$b$	$\{a, b\}$	$b$	$\{b, c\}$
$c$	$G$	$\{b, c\}$	$c$

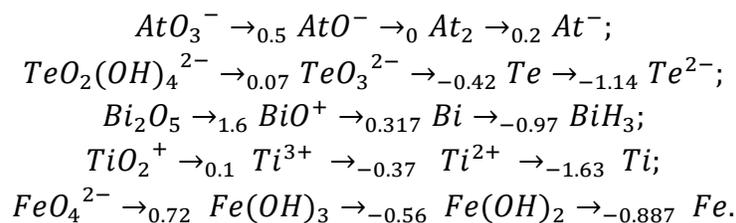
Then  $(G, \circ)$  is a commutative hypergroup.

**Definition 2.4.** [7] Two hypergroupoids  $(H, \circ)$  and  $(K, *)$  are said to be isomorphic hypergroupoids, written as  $H \cong K$ , if there exists a bijective function  $f: H \rightarrow K$  such that  $f(x \circ y) = f(x) * f(y)$  for all  $x, y \in H$ .

### 3. STANDARD REDUCTION POTENTIALS FOR FOUR OXIDATION STATES OF ELEMENTS

Through the Latimer diagrams of all elements, we select a lot of chemical species that were recorded four consecutive standard reduction potentials in acidic and/or basic solution [19].

**Example 3.** The following Latimer diagram are for Astatine, Tellurium, Bismuth, Titanium and Iron respectively that are elements with four consecutive standard reduction potentials in acidic and/or basic solution.



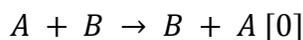
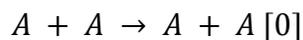
The redox reactions for Astatine, Tellurium and Bismuth were studied as chemical hyperstructures by the authors [2] in which the hyperstructures of Bismuth and Tellurium were proved to be isomorphic. Let  $D, C, B$  and  $A$  be chemical species of an arbitrary element  $U$  and let  $x, y, z$  and  $w$  be the number of equivalents oxidized or reduced of  $D, C, B$  and  $A$  respectively with  $x > y > z > w$ . Let  $n_1 = x - y$ ,  $n_2 = y - z$  and  $n_3 = z - w$  be the electronic differences and let  $\alpha, \beta$  and  $\gamma$  be the potential differences (in volt) between  $D$  and  $C$ ,  $C$  and  $B$ , and  $B$  and  $A$  respectively. Given the Latimer diagram of  $U$  as  $D \xrightarrow{\alpha} C \xrightarrow{\beta} B \xrightarrow{\gamma} A$ , we present the following reductions:

$$\begin{aligned} (1) \quad D &\rightarrow B, E_1 = \frac{\alpha n_1 + \beta n_2}{n_1 + n_2}, \\ (2) \quad D &\rightarrow A, E_2 = \frac{\alpha n_1 + \beta n_2 + \gamma n_3}{n_1 + n_2 + n_3}, \\ (3) \quad C &\rightarrow A, E_3 = \frac{\beta n_2 + \gamma n_3}{n_2 + n_3}. \end{aligned}$$

Here,  $E_1, E_2, E_3$  are the potential differences (in volt) of the entries that connects  $D$  to  $B$ ,  $D$  to  $A$ , and  $C$  to  $A$  respectively. In our paper, we are concerned about the chemical hyperstructures of  $U$  under the condition  $\alpha > \beta > \gamma$ . One can easily see that  $\beta < E_1 < \alpha, \gamma < E_2 < \alpha, \gamma < E_3 < \beta, E_3 < E_1$ .

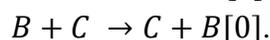
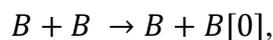
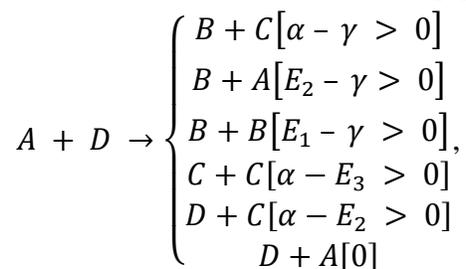
Let  $\{x, x', y, y'\} \subseteq \{A, B, C, D\}$  such that  $x \xrightarrow{a} x'$  and  $y \xrightarrow{b} y'$  where  $a, b$  are potential differences. We get the following redox reaction:  $x + y \xrightarrow{a+b} x' + y'$ . If  $E = a + b > 0$  then our redox reaction is spontaneous. Otherwise, it is not spontaneous.

We consider the set  $H = \{A, B, C, D\}$ . The following are all possible spontaneous redox combinations for  $H$ .



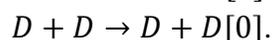
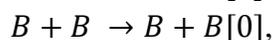
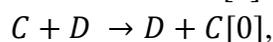
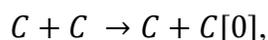
For the spontaneous reactions of  $A + C$ , we consider two cases: Case  $E_2 < \beta$  and Case  $E_2 \geq \beta$ .

- Case  $E_2 < \beta$ . We get that  $A + C \rightarrow \begin{cases} B + B[\beta - \gamma > 0] \\ C + B[-E_3 + \beta > 0] \\ D + B[-E_2 + \beta > 0] \\ C + A[0] \end{cases}$ .
- Case  $E_2 \geq \beta$ . We get that  $A + C \rightarrow \begin{cases} B + B[\beta - \gamma > 0] \\ C + B[-E_3 + \beta > 0], \\ C + A[0] \end{cases}$ .



For the spontaneous reactions of  $B + D$ , we consider two cases: Case  $E_2 \leq \beta$  and Case  $E_2 > \beta$ .

- Case  $E_2 \leq \beta$ . We get that  $B + D \rightarrow \begin{cases} C + C[\alpha - \beta > 0] \\ D + C[\alpha - E_1 > 0] \\ C + B[E_1 - \beta > 0] \\ D + B[0] \end{cases}$ .
- Case  $E_2 > \beta$ . We get that  $B + D \rightarrow \begin{cases} C + C[\alpha - \beta > 0] \\ D + C[\alpha - E_1 > 0] \\ C + B[E_1 - \beta > 0], \\ C + A[E_2 - \beta > 0] \\ D + B[0] \end{cases}$ .



**Remark 1.** The conditions  $E_2 < \beta$ ,  $E_2 > \beta$  and  $E_2 = \beta$  can be written as  $\frac{\alpha n_1 + \gamma n_3}{n_1 + n_3} < \beta$ ,  $\frac{\alpha n_1 + \gamma n_3}{n_1 + n_3} > \beta$  and  $\frac{\alpha n_1 + \gamma n_3}{n_1 + n_3} = \beta$  respectively.

For all  $x, y \in H$ , we define “ $\oplus_1$ ”, “ $\oplus_2$ ” and “ $\oplus_3$ ” on  $H$  for  $E_2 < \beta$ ,  $E_2 > \beta$  and  $E_2 = \beta$  respectively. Where  $x \oplus_1 y = z$ ,  $x \oplus_2 y = z'$  and  $x \oplus_3 y = z^*$ , and  $z, z', z^*$  are in the product of the spontaneous redox reaction with greatest potential difference that occurs between  $x$  and  $y$ . Then we obtain Table 3 for  $(H, \oplus_1)$  which is the same as that for  $(H, \oplus_2)$  and  $(H, \oplus_3)$ .

**Table 3:**  $(H, \oplus_1)$ .

$\oplus_1$	$A$	$B$	$C$	$D$
$A$	$A$	$\{A, B\}$	$B$	$\{B, C\}$
$B$	$\{A, B\}$	$B$	$\{B, C\}$	$C$
$C$	$B$	$\{B, C\}$	$C$	$\{C, D\}$
$D$	$\{B, C\}$	$C$	$\{C, D\}$	$D$

**Theorem 3.1.**  $(H, \oplus_1)$ ,  $(H, \oplus_2)$  and  $(H, \oplus_3)$  are commutative  $H_v$ -semigroups.

**Proof.**  $(H, \oplus_1)$  is isomorphic to the  $H_v$ -semigroups of Tellurium and Bismuth presented in [2]. ■

Let  $H = \{A, B, C, D\}$  and define “ $\otimes_1$ ”, “ $\otimes_2$ ” and “ $\otimes_3$ ” on  $H$  for  $E_2 < \beta$ ,  $E_2 > \beta$  and  $E_2 = \beta$  respectively. Where  $x \otimes_1 y = z$ ,  $x \otimes_2 y = z'$  and  $x \otimes_3 y = z^*$ , and  $z, z', z^*$  are in the product of any spontaneous redox reaction that occurs between  $x$  and  $y$ . Then we obtain Table 4 for  $(H, \otimes_1)$ , Table 5 for  $(H, \otimes_2)$  and Table 6 for  $(H, \otimes_3)$ .

**Table 4:**  $(H, \otimes_1)$ .

$\otimes_1$	$A$	$B$	$C$	$D$
$A$	$A$	$\{A, B\}$	$H$	$H$
$B$	$\{A, B\}$	$B$	$\{B, C\}$	$\{B, C, D\}$
$C$	$H$	$\{B, C\}$	$C$	$\{C, D\}$
$D$	$H$	$\{B, C, D\}$	$\{C, D\}$	$D$

**Table 5:**  $(H, \otimes_2)$ .

$\otimes_2$	$A$	$B$	$C$	$D$
$A$	$A$	$\{A, B\}$	$\{A, B, C\}$	$H$
$B$	$\{A, B\}$	$B$	$\{B, C\}$	$H$
$C$	$\{A, B, C\}$	$\{B, C\}$	$C$	$\{C, D\}$
$D$	$H$	$H$	$\{C, D\}$	$D$

**Table 6:**  $(H, \otimes_3)$ .

$\otimes_3$	A	B	C	D
A	A	{A, B}	{A, B, C}	H
B	{A, B}	B	{B, C}	{B, C, D}
C	{A, B, C}	{B, C}	C	{C, D}
D	H	{B, C, D}	{C, D}	D

**Theorem 3.2.**  $(H, \otimes_1)$  is a commutative hypergroup.

**Proof.**  $(H, \otimes_1)$  is isomorphic to the hypergroup of Tellurium presented in [2]. ■

**Theorem 3.3.**  $(H, \otimes_2)$  is a commutative hypergroup.

**Proof.** We prove that  $(H, \otimes_2)$  is isomorphic to  $(H, \otimes_1)$ . Let  $f : (H, \otimes_1) \rightarrow (H, \otimes_2)$  be a bijection function defined by:  $f(A) = D, f(B) = C, f(C) = B$  and  $f(D) = A$ . Then

$$f(x \otimes_1 y) = \begin{cases} \{f(x), f(y)\} & \text{if } x, y \in \{A, B\} \text{ or } x, y \in \{B, C\} \text{ or } x, y \in \{C, D\}; \\ \{A, B, C\} & \text{if } x = B \text{ and } y = D; \\ H & \text{otherwise.} \end{cases}$$

$$= f(x) \otimes_2 f(y).$$

**Theorem 3.4.**  $(H, \otimes_3)$  is a commutative hypergroup.

**Proof.** It is clear that  $(H, \otimes_3)$  is a commutative quasihypergroup. We need to show that  $(H, \otimes_3)$  is a semihypergroup, i.e., we need to show that  $(H, \otimes_3)$  is associative.

Since  $(\{A, B, C\}, \otimes_3)$  and  $(\{B, C, D\}, \otimes_3)$  are isomorphic hypergroups (isomorphic to the hypergroup presented in Example 2), it suffices to consider the following cases for  $y, z \in H$ .

$A \otimes_3 (D \otimes_3 z) \supseteq A \otimes_3 D = H$	$(A \otimes_3 D) \otimes_3 z = H \otimes_3 z = H$
$A \otimes_3 (y \otimes_3 D) \supseteq A \otimes_3 D = H$	$(A \otimes_3 y) \otimes_3 D \supseteq A \otimes_3 D = H$
$B \otimes_3 (A \otimes_3 D) \supseteq B \otimes_3 H = H$	$(B \otimes_3 A) \otimes_3 D = H$
$B \otimes_3 (D \otimes_3 A) = H$	$(B \otimes_3 D) \otimes_3 A = H$
$C \otimes_3 (A \otimes_3 D) = H$	$(C \otimes_3 A) \otimes_3 D = H$
$C \otimes_3 (D \otimes_3 A) = H$	$(C \otimes_3 D) \otimes_3 A \supseteq D \otimes_3 A = H$
$D \otimes_3 (A \otimes_3 z) \supseteq D \otimes_3 A = H$	$(D \otimes_3 A) \otimes_3 z = H \otimes_3 z = H$
$D \otimes_3 (y \otimes_3 A) \supseteq D \otimes_3 A = H$	$(D \otimes_3 y) \otimes_3 A \supseteq D \otimes_3 A = H$

Therefore,  $(H, \otimes_3)$  is a commutative hypergroup. ■

**Remark 2.**  $(H, \otimes_3)$  is not isomorphic to neither  $(H, \otimes_1)$  nor to  $(H, \otimes_2)$ .

Next, we present some chemical interpretation for our results.

**Remark 3.** From the tables of  $(H, \otimes_1)$ ,  $(H, \otimes_2)$  and  $(H, \otimes_3)$ , we deduce that  $B$  and  $C$  are most likely to occur in nature and  $D$  is the least likely to occur in nature.

**Remark 4.** Having each element  $x$  in:  $(H, \otimes_1)$ ,  $(H, \otimes_2)$  and  $(H, \otimes_3)$  an identity, implies that  $y \in H$  is present in the product of at least one of the spontaneous reactions that occurs between  $x$  and  $y$ .

#### 4. CHEMICAL INTERPRETATIONS AND EXAMPLES

Through the Latimer diagrams of all elements, we select a lot of chemical species that were recorded four decreasing consecutive standard reduction potentials in acidic and/or basic solution [19]. In this section, we present some of them and identify their chemical hyperstructures. We present Silver and Antimony as examples when  $E_2 < \beta$ , Iron and Platinum as examples when  $E_2 > \beta$  and Americium when  $E_2 = \beta$ .

First, we present some examples when  $E_2 < \beta$ .

**Example 4.** The chemical hyperstructures for Titanium ( $Ti$ ) in [5], Bismuth ( $Bi$ ) and Tellurium ( $Te$ ) in [2] are examples of chemical hyperstructures when  $\alpha > \beta > \gamma$  and  $E_2 < \beta$ .

**Example 5.** Silver, denoted by  $Ag$ , is a soft and white transition metal that exhibits the highest electrical conductivity, thermal conductivity, and reflectivity of any metal. It is a member of group 11 of the periodic table. The Latimer diagram of silver in acidic solution is given  $AgO^+ \rightarrow_{2.1} Ag^{2+} \rightarrow_{1.98} Ag^+ \rightarrow_{0.799} Ag$ . We have that  $\alpha = 2.1$ ,  $\beta = 1.98$ ,  $\gamma = 0.799$  and  $n_1 = n_2 = n_3 = 1$ . Since  $E_2 = \frac{2.1+1.98+0.799}{3} = 1.626 < \beta$ , it follows that:

- $(H = \{Ag, Ag^+, Ag^{2+}, Ag^{3+}\}, \oplus_1)$  is a commutative  $H_v$ -semigroup with Table 7:

**Table 7:**  $(H = \{Ag, Ag^+, Ag^{2+}, Ag^{3+}\}, \oplus_1)$ .

$\oplus_1$	$Ag$	$Ag^+$	$Ag^{2+}$	$Ag^{3+}$
$Ag$	$Ag$	$\{Ag, Ag^+\}$	$Ag^+$	$\{Ag^+, Ag^{2+}\}$
$Ag^+$	$\{Ag, Ag^+\}$	$Ag^+$	$\{Ag^+, Ag^{2+}\}$	$Ag^{2+}$
$Ag^{2+}$	$Ag^+$	$\{Ag^+, Ag^{2+}\}$	$Ag^{2+}$	$\{Ag^{2+}, Ag^{3+}\}$
$Ag^{3+}$	$\{Ag^+, Ag^{2+}\}$	$Ag^{2+}$	$\{Ag^{2+}, Ag^{3+}\}$	$Ag^{3+}$

- ( $H = \{Ag, Ag^+, Ag^{2+}, Ag^{3+}\}, \otimes_1$ ) is a commutative hypergroup with Table 8:

**Table 8:** ( $H = \{Ag, Ag^+, Ag^{2+}, Ag^{3+}\}, \otimes_1$ ).

$\otimes_1$	$Ag$	$Ag^+$	$Ag^{2+}$	$Ag^{3+}$
$Ag$	$Ag$	$\{Ag, Ag^+\}$	$H$	$H$
$Ag^+$	$\{Ag, Ag^+\}$	$Ag^+$	$\{Ag^+, Ag^{2+}\}$	$\{Ag^+, Ag^{2+}, Ag^{3+}\}$
$Ag^{2+}$	$H$	$\{Ag^+, Ag^{2+}\}$	$Ag^{2+}$	$\{Ag^{2+}, Ag^{3+}\}$
$Ag^{3+}$	$H$	$\{Ag^+, Ag^{2+}, Ag^{3+}\}$	$\{Ag^{2+}, Ag^{3+}\}$	$Ag^{3+}$

**Example 6.** Antimony, denoted by  $Sb$ , is a lustrous gray metalloid and it is a member of group 15 of the periodic table in accordance with periodic trends, it is more electronegative than bismuth, and less electronegative than tellurium. The Latimer diagram of Antimony in acidic solution is given as  $Sb_2O_5 \xrightarrow{0.605} SbO^+ \xrightarrow{0.204} Sb \xrightarrow{-0.510} SbH_3$ .

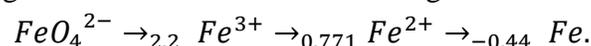
We have that  $\alpha = 0.605, \beta = 0.204, \gamma = -0.510$  and  $n_1 = 2, n_2 = 3, n_3 = 3$ . Since  $E_2 = \frac{0.605(2)+0.204(3)-0.510(3)}{8} = 0.0365 < \beta$ , it follows that ( $H = \{Sb^{3-}, Sb, Sb^{3+}, Sb^{5+}\}, \otimes_1$ ) is a hypergroup with Table 9:

**Table 9:** ( $H = \{Sb^{3-}, Sb, Sb^{3+}, Sb^{5+}\}, \otimes_1$ ).

$\otimes_1$	$Sb^{3-}$	$Sb$	$Sb^{3+}$	$Sb^{5+}$
$Sb^{3-}$	$Sb^{3-}$	$\{Sb^{3-}, Sb\}$	$H$	$H$
$Sb$	$\{Sb^{3-}, Sb\}$	$Sb$	$\{Sb, Sb^{3+}\}$	$\{Sb, Sb^{3+}, Sb^{5+}\}$
$Sb^{3+}$	$H$	$\{Sb, Sb^{3+}\}$	$Sb^{3+}$	$\{Sb^{3+}, Sb^{5+}\}$
$Sb^{5+}$	$H$	$\{Sb, Sb^{3+}, Sb^{5+}\}$	$\{Sb^{3+}, Sb^{5+}\}$	$Sb^{5+}$

According to Table 9, we can deduce that  $Sb^{3+}$  is the most common oxidation state. Next, we present two examples when  $E_2 > \beta$ .

**Example 7.** Iron, denoted by  $Fe$ , is a metal in the first transition series and it is the fourth most common element in the Earth's crust. Iron exists in a wide range of oxidation states, +2 to +7, with +2 and +3 are the most common. It is a member of group 8 the periodic table. The Latimer diagram of iron in acidic solution is given as follows:

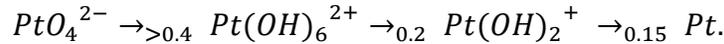


We have that  $\alpha = 2.2, \beta = 0.771, \gamma = -0.44$  and  $n_1 = 3, n_2 = 1, n_3 = 2$ . Since  $E_2 = \frac{2.2(3)+0.771-0.44(2)}{6} = 1.082 > \beta$ , ( $H = \{Fe, Fe^{2+}, Fe^{3+}, Fe^{6+}\}, \otimes_2$ ) is a hypergroup with Table 10:

**Table 10:**  $(H = \{Fe, Fe^{2+}, Fe^{3+}, Fe^{6+}\}, \otimes_2)$ .

$\otimes_2$	$Fe$	$Fe^{2+}$	$Fe^{3+}$	$Fe^{6+}$
$Fe$	$Fe$	$\{Fe, Fe^{2+}\}$	$\{Fe, Fe^{2+}, Fe^{3+}\}$	$H$
$Fe^{2+}$	$\{Fe, Fe^{2+}\}$	$Fe^{2+}$	$\{Fe^{2+}, Fe^{3+}\}$	$H$
$Fe^{3+}$	$\{Fe, Fe^{2+}, Fe^{3+}\}$	$\{Fe^{2+}, Fe^{3+}\}$	$Fe^{3+}$	$\{Fe^{3+}, Fe^{6+}\}$
$Fe^{6+}$	$H$	$H$	$\{Fe^{3+}, Fe^{6+}\}$	$Fe^{6+}$

**Example 8.** Platinum, denoted by  $Pt$ , is a dense, malleable, ductile, highly unreactive, precious white transition metal. It is a member of the platinum group of elements and group 10 of the periodic table of elements. It is one of the rarer elements in Earth's crust and is one of the least reactive metals. It has remarkable resistance to corrosion, even at high temperatures, and is therefore considered a noble metal. The most common oxidation states of platinum are +2 and +4. The Latimer diagram of Platinum in basic solution is given as



We have that  $\alpha > 0.4$ ,  $\beta = 0.2$ ,  $\gamma = 0.15$  and  $n_1 = n_2 = n_3 = 2$ . Since  $E_2 > \frac{0.4(2)+0.2(2)+0.15(2)}{6} = 0.25 > \beta$ , it follows that  $(H = \{Pt, Pt^{2+}, Pt^{4+}, Pt^{6+}\}, \otimes_2)$  is a hypergroup with Table 11:

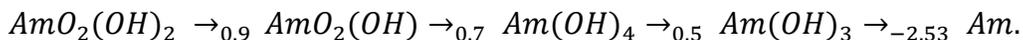
**Table 11:**  $(H = \{Pt, Pt^{2+}, Pt^{4+}, Pt^{6+}\}, \otimes_2)$ .

$\otimes_2$	$Pt$	$Pt^{2+}$	$Pt^{4+}$	$Pt^{6+}$
$Pt$	$Pt$	$\{Pt, Pt^{2+}\}$	$\{Pt, Pt^{2+}, Pt^{4+}\}$	$H$
$Pt^{2+}$	$\{Pt, Pt^{2+}\}$	$Pt^{2+}$	$\{Pt^{2+}, Pt^{4+}\}$	$H$
$Pt^{4+}$	$\{Pt, Pt^{2+}, Pt^{4+}\}$	$\{Pt^{2+}, Pt^{4+}\}$	$Pt^{4+}$	$\{Pt^{4+}, Pt^{6+}\}$
$Pt^{6+}$	$H$	$H$	$\{Pt^{4+}, Pt^{6+}\}$	$Pt^{6+}$

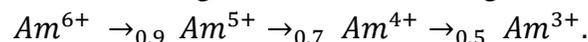
According to Table 11, we can deduce that  $Pt^{2+}$  and  $Pt^{4+}$  are the most common oxidation states.

Finally, we present an example when  $E_2 = \beta$ .

**Example 9.** Americium, denoted by  $Am$ , is a radioactive chemical element. It is a transuranic member of the actinide series, in the periodic table located under the lanthanide element europium. Americium is a relatively soft radioactive metal with silvery appearance. The Latimer diagram of Americium in basic solution is given as follows:



By constructing again the Latimer diagram without Am, we get the following diagram:



Let  $H = \{Am^{3+}, Am^{4+}, Am^{5+}, Am^{6+}\}$ . Having  $\alpha = 0.9 > \beta = 0.7 > \gamma = 0.5$  and  $n_1 = n_2 = n_3 = 1$  implies that  $E_2 = \frac{0.9+0.7+0.5}{4} = 0.7 = \beta$ . Thus,  $(H = \{Am^{3+}, Am^{4+}, Am^{5+}, Am^{6+}\}, \otimes_3)$  is a commutative hypergroup with Table 12:

**Table 12:**  $(H = \{Am^{3+}, Am^{4+}, Am^{5+}, Am^{6+}\}, \otimes_3)$ .

$\otimes_3$	$Am^{3+}$	$Am^{4+}$	$Am^{5+}$	$Am^{6+}$
$Am^{3+}$	$Am^{3+}$	$\{Am^{3+}, Am^{4+}\}$	$H \setminus \{Am^{6+}\}$	$H$
$Am^{4+}$	$\{Am^{3+}, Am^{4+}\}$	$Am^{4+}$	$\{Am^{4+}, Am^{5+}\}$	$H \setminus \{Am^{3+}\}$
$Am^{5+}$	$H \setminus \{Am^{6+}\}$	$\{Am^{4+}, Am^{5+}\}$	$Am^{5+}$	$\{Am^{5+}, Am^{6+}\}$
$Am^{6+}$	$H$	$H \setminus \{Am^{3+}\}$	$\{Am^{5+}, Am^{6+}\}$	$Am^{6+}$

## 5. CONCLUSION

Mathematics is used widely in chemistry as well as all other sciences. Mathematical chemistry is the area of research engaged in novel applications of mathematics to chemistry; it concerns itself principally with the mathematical modeling of chemical phenomena. Chemical reactions are examples of the phenomena when composition of two or more elements is a non-empty set of elements.

This study focused on the relationship between algebraic hyperstructures and chemistry. The goal of this paper is to help mathematics by offering suggestions about how algebraic hyperstructures and chemistry can be coordinated better in curriculum. More precisely, it dealt with two non-isomorphic hyperstructures of arbitrary elements with four different oxidation states under the condition  $\alpha > \beta > \gamma$ .

For future work, it will be interesting to generalize our work and investigate all algebraic hyperstructures for chemical reactions for four oxidation states of an arbitrary element by covering all the remaining conditions for  $\alpha, \beta, \gamma$ .

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