

Chemical Salt Reactions as Algebraic Hyperstructures

DARIUSH HEIDARI^{1,*}, DAVOOD MAZAHERI¹ and BIJAN DAVVAZ²

¹ Faculty of science, Mahallat Institute of Higher Education, Mahallat, Iran

¹ Faculty of Engineering, Mahallat Institute of Higher Education, Mahallat, Iran

² Department of Mathematics, Yazd University, Yazd, Iran

ARTICLE INFO

Article History:

Received 11 January 2018

Accepted 22 January 2018

Published online 30 July 2019

Academic Editor: Ivan Gutman

Keywords:

Hyperstructures

Semihypergroup

H_v -semigroup

Salt reaction

ABSTRACT

A salt metathesis reaction is a chemical process involving the exchange of bonds between two reacting chemical species, which results in the creation of products with similar or identical bonding affiliations. Hyperstructure theory is studied from the theoretical point of view and for its applications. In this paper, we provide some examples of hyperstructures associated with salt metathesis reactions, and we observe that these chemical reactions are examples of the phenomena when composition of two elements is a set of elements.

© 2019 University of Kashan Press. All rights reserved

1. INTRODUCTION

Hyperstructure theory, introduced in 1934 by F. Marty [13], is studied from the theoretical point of view and for its applications to many subjects of pure and applied mathematics, see [4, 12]. Algebraic hyperstructures are a generalization of classical algebraic structures. In a classical algebraic structure the composition of two elements is an element, while in an algebraic hyperstructure the composition of two elements is a non-empty set. One of motivations for the study of hyperstructures comes from chemical reactions. In [6], Davvaz and Dehghan-Nezhad provided examples of hyperstructures associated with chain reactions. In [8], Davvaz et al. introduced examples of weak hyperstructures associated with dismutation reactions. In [11], Davvaz et al. investigated the examples of hyperstructures and weak hyperstructures associated with redox reactions. Also, see [1, 2, 5, 10]. In [3], Chung et al. investigated mathematical structures of chemical reactions for three consecutive oxidation states of elements.

* Corresponding Author (Email address: dheidari82@gmail.com).

DOI: 10.22052/ijmc.2018.114473.1339

Let H be a non-empty set. Then the map $\circ: H \times H \rightarrow P^*(H)$ is called a hyper operation when $P^*(H)$ is the family of non-empty subsets of H . The couple (H, \circ) is called a hypergroupoid. The hyper product of two subsets A and B of H defines as follows

$$A \circ B = \bigcup_{a \in A, b \in B} a \circ b; \quad x \circ A = \{x\} \circ A \text{ and } A \circ x = A \circ \{x\}.$$

The theory of H_v -structures has been introduced by Vougiouklis [17]. The concept of H_v -structures constitutes a generalization of the well-known algebraic hyperstructures (hypergroups, hyperrings, hypermodules). Actually, some axioms concerning the above hyperstructures are replaced by their corresponding weak axioms. Basic definitions and results about the H_v -structures and their applications can be found in [7, 15, 16].

Definition 1.1. The hypergroupoid (H, \circ) is called

- 1 an H_v -semigroup if the *weak associativity* property holds that is for every $x, y, z \in H$ we have $x \circ (y \circ z) \cap (x \circ y) \circ z \neq \emptyset$;
- 2 a *semihypergroup* if the *associativity* property holds that is for every $x, y, z \in H$ we have $x \circ (y \circ z) = (x \circ y) \circ z$;
- 3 a *quasihypergroup* if *reproductive* axiom holds that is for every $x \in H$ we have $x \circ H = H = H \circ x$;
- 4 an H_v -group if it is an H_v -semigroup and quasihypergroup;
- 5 a *hypergroup* if it is a semihypergroup and quasihypergroup;
- 6 a *commutative hypergroupoid* if for every $x, y \in H$ we have $x \circ y = y \circ x$.

2. HYPERSTRUCTURES OF SALT REACTIONS

In a salt metathesis reaction, cations and anions exchange partners. This reaction usually takes place in aqueous solutions. Metathesis reaction is a type of chemical reactions, which include combination, decomposition, and displacement. When a soluble salt (like sodium chloride) is dissolved in water, it decomposes and becomes hydrated ions. If you pour two solutions of different electrolytes together, the mixture will have all ions from the two electrolytes. Ions of the same charge usually repel each other, but ions of opposite charge may form a stable molecule or solid. Cations of one electrolyte meet anions of the other. If they form a more stable substance such as a solid or neutral molecules, exchange or metathesis reaction takes place. The new couples form a precipitation, gas, or neutral molecules. (See [14] for more details.)

In the following lemma we construct a semihypergroup by a non-empty set of atoms X and an associative hyper operation \circ on $X \times X$. The semihypergroup $(X \times X, \circ)$ is denoted by $\mathfrak{S}[X]$.

Lemma 2.1. *Let X be a non-empty set and for every $(x_1, x_2), (y_1, y_2) \in X \times X$ define $(x_1, x_2) \circ (y_1, y_2) = \{(z, z) | z \in \{x_1, x_2, y_1, y_2\}\}$. Then, $\mathfrak{S}[X] = (X \times X, \circ)$ is a semihypergroup.*

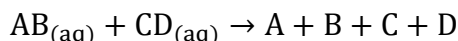
Proof. Suppose that $a_i = (x_i, y_i) \in X \times X$, where $i = 1, 2, 3$. Then,

$$a_1 \circ (a_2 \circ a_3) = \{(x_i, x_i), (y_i, y_i) | 1 \leq i \leq 3\} = (a_1 \circ a_2) \circ a_3.$$

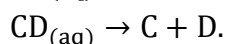
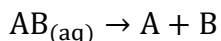
Therefore, $\mathfrak{S}[X]$ is a semihypergroup.

From now on, let A and C be cations, B and D be anions and AB, AD, CB and CD be salts. Based on the solubility of the reacting and produced salts we can consider the following four cases:

Case 1. *All ions and salts are soluble in water.* Then the overall reaction is



This reaction takes place in following steps:



All ions formed from these salts are soluble in water, consequently, all of them are bystander ions and no reaction takes place and we obtain the following hyper operation table:

Table 1: $AB + CD \rightarrow A + B + C + D$

\oplus	A	B	C	D	AB	CD
A	A	A,B	A,C	A,D	A,B	A,C,D
B	A,B	B	B,C	B,D	A,B	B,C,D
C	A,C	B,C	C	C,D	A,B,C	C,D
D	A,D	B,D	C,D	D	A,B,D	C,D
AB	A,B	A,B	A,B,C	A,B,D	A,B	A,B,C,D
CD	A,C,D	B,C,D	C,D	C,D	A,B,C,D	C,D

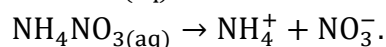
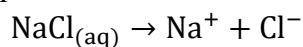
Theorem 2.2. *Let $H := \{A, B, C, D, AB, CD\}$, where A and C are cations and B and D are anions. Let $x \oplus y$ be the chemical interaction of x and y for every $x, y \in H$. Then, (H, \oplus) is a semihypergroup.*

Proof. Let $X = \{A, B, C, D\}$. Then, $\mathfrak{S}[X]$ is a semihypergroup by Lemma 2.1 and the mapping $\varphi: (H, \oplus) \rightarrow \mathfrak{S}[X]$ defined by

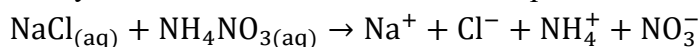
$$\varphi(z) = \begin{cases} (z, z) & z \in X \\ (A, B) & z = AB \\ (C, D) & z = CD \end{cases}$$

is a monomorphism. Therefore, (H, \oplus) is isomorphic to a sub-semihypergroup of $\mathfrak{S}[X]$ and the proof is complete.

Example 1. What will be the reaction of sodium chloride and ammonium nitrate? These two salts are soluble and decompose to their ions.



Since $\text{Na}^+, \text{Cl}^-, \text{NH}_4^+$ and NO_3^- formed from these salts are soluble in water, then all of them are bystander ions and no reaction takes place:

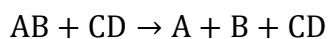


Then we have the following commutative H_v -semigroup:

Table 2: $\text{NaCl}_{(\text{aq})} + \text{NH}_4\text{NO}_3(\text{aq}) \rightarrow \text{Na}^+ + \text{Cl}^- + \text{NH}_4^+ + \text{NO}_3^-$

\oplus	Na^+	Cl^-	NH_4^+	NO_3^-	NaCl	NH_4NO_3
Na^+	Na^+	Na^+ Cl^-	Na^+ NH_4^+	Na^+ NO_3^-	Na^+ Cl^-	Na^+ NH_4^+ NO_3^-
Cl^-	Na^+ Cl^-	Cl^-	Cl^- NH_4^+	Cl^- NO_3^-	Na^+ Cl^-	Cl^- NH_4^+ NO_3^-
NH_4^+	Na^+ NH_4^+	Cl^- NH_4^+	NH_4^+	NH_4^+ NO_3^-	NH_4^+ Na^+ Cl^-	NH_4^+ NO_3^-
NO_3^-	Na^+ NO_3^-	Cl^- NO_3^-	NH_4^+ NO_3^-	NO_3^-	NO_3^- Na^+ Cl^-	NO_3^- NH_4^+
NaCl	Na^+ Cl^-	Na^+ Cl^-	NH_4^+ Na^+ Cl^-	NO_3^- Na^+ Cl^-	Na^+ Cl^-	Na^+ Cl^- NO_3^- NH_4^+
NH_4NO_3	Na^+ NH_4^+ NO_3^-	Cl^- NH_4^+ NO_3^-	NH_4^+ NO_3^-	NO_3^- NH_4^+	Na^+ Cl^- NO_3^- NH_4^+	NO_3^- NH_4^+

Case 2. On of the reacting salts is soluble in water and the other is insoluble in water. Let AB be soluble and CD be insoluble. Then the reaction is



and we obtain the following hyper operation table:

Table 3: $AB + CD \rightarrow A + B + CD$

\oplus	A	B	AB	CD
A	A	A,B	A,B	A,CD
B	A,B	B	A,B	B,CD
AB	A,B	A,B	A,B	A,B,CD
CD	A,CD	B,CD	A,B,CD	CD

Theorem 2.3. Let $H := \{A, B, AB, CD\}$, where A is a cation and B is an anion and $x \oplus y$ is the chemical interaction of x and y for every $x, y \in H$. Then (H, \oplus) is a semihypergroup.

Proof. Similar to the proof of Theorem 2.2 we conclude that \oplus is associative on $\{A, B, AB\}$. On the other hand for every $X, Y \in H$ we have

$$CD \oplus (X \oplus Y) = \{CD\} \cup X \oplus Y = (CD \oplus X) \oplus Y.$$

Therefore, the result holds.

Example 2. Sodium nitrate is soluble and silver chloride is not soluble in water

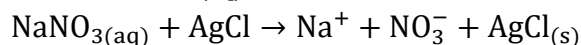
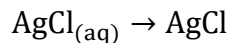
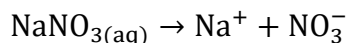
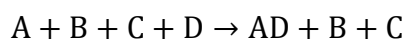
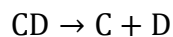
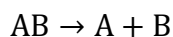


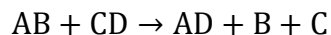
Table 4: $\text{NaNO}_{3(\text{aq})} + \text{AgCl} \rightarrow \text{Na}^+ + \text{NO}_3^- + \text{AgCl}_{(\text{s})}$

\oplus	Na^+	NO_3^-	NaNO_3	AgCl
Na^+	Na^+	Na^+ NO_3^-	Na^+ NO_3^-	Na^+ AgCl
NO_3^-	Na^+ NO_3^-	NO_3^-	Na^+ NO_3^-	NO_3^- AgCl
NaNO_3	Na^+ NO_3^-	Na^+ NO_3^-	Na^+ NO_3^-	Na^+ NO_3^- AgCl
AgCl	Na^+ AgCl	NO_3^- AgCl	Na^+ NO_3^- AgCl	AgCl

Case 3. Both Salts are soluble in water but one anion-cation pair forms a solid and the other pair is soluble in water and remains in ion form. Let AB and CD be soluble and AD be insoluble solid produced from the reaction. Then the reaction steps are



and the overall reaction is



and we obtain the following hyper operation table:

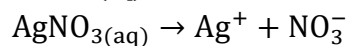
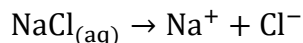
Table 5: $AB + CD \rightarrow AD + B + C$

\oplus	A	B	C	D	AB	CD	AD
A	A	A,B	A,C	AD	A,B	C,AD	A,AD
B	A,B	B	B,C	B,D	A,B	B,C,D	B,AD
C	A,C	B,C	C	C,D	C,A,B	C,D	C,AD
D	AD	B,D	C,D	D	B,AD	D,C	D,AD
AB	A,B	A,B	C,A,B	B,AD	A,B	AD,B,C	B,A,AD
CD	AD,C	B,C,D	C,D	C,D	AD,B,C	C,D	C,D,AD
AD	A,AD	AD,B	AD,C	AD,D	A,B,AD	C,D,AD	AD

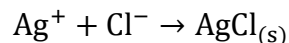
Theorem 2.4. Let $H := \{A, B, C, D, AB, CD, AD\}$, where A and C are cations and B and D are anions and $x \oplus y$ is the chemical interaction of x and y for every $x, y \in H$. Then (H, \oplus) is an H_v -semigroup.

Proof. Suppose that $x, y, z \in H$. If $\{x, y, z\} \subseteq \{A, B, C, AB\}$ or $\{x, y, z\} \subseteq \{B, C, D, CD\}$ then $x \oplus (y \oplus z) = (x \oplus y) \oplus z$ since, $(\{A, B, C, AB\}, \oplus)$ and $(\{B, C, D, CD\}, \oplus)$ are isomorphic to a sub-semihypergroup of $\mathfrak{S}[\{A, B, C\}]$ and $\mathfrak{S}[\{B, C, D\}]$, respectively. Otherwise, $AD \in x \oplus (y \oplus z) \cap (x \oplus y) \oplus z$. So, (H, \oplus) is an H_v -semigroup.

Example 3. Sodium chloride and silver nitrate are soluble salts in water, and decompose to their constituent ions:



When the silver ions and chloride ions meet in solution, they combine and form a solid, which appears as a white precipitate:



Sodium and nitrate ions do not react (because sodium nitrate is a soluble salt) and they are called bystander (or spectator) ions. In result, we can write the overall reaction as:

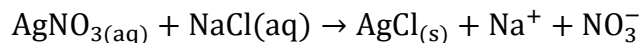
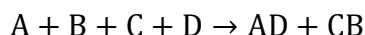
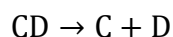
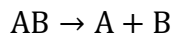


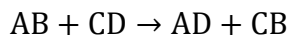
Table 6: $\text{AgNO}_3(\text{aq}) + \text{NaCl}(\text{aq}) \rightarrow \text{AgCl}(\text{s}) + \text{Na}^+ + \text{NO}_3^-$

\oplus	Ag^+	NO_3^-	Na^+	Cl^-	AgNO_3	NaCl	AgCl
Ag^+	Ag^+	Ag^+ NO_3^-	Ag^+ Na^+	AgCl	Ag^+ NO_3^-	Na^+ AgCl	Ag^+ AgCl
NO_3^-	Ag^+ NO_3^-	NO_3^-	NO_3^- Na^+	NO_3^- Cl^-	NO_3^- Ag^+	NO_3^- Na^+ Cl^-	NO_3^- AgCl
Na^+	Ag^+ Na^+	NO_3^- Na^+	Na^+	Na^+ Cl^-	Na^+ Ag^+ NO_3^-	Na^+ Cl^-	Na^+ AgCl
Cl^-	AgCl	NO_3^- Cl^-	Na^+ Cl^-	Cl^-	NO_3^- AgCl	Cl^- Na^+	Cl^- AgCl
AgNO_3	Ag^+ NO_3^-	NO_3^- Ag^+	Na^+ Ag^+ NO_3^-	NO_3^- AgCl	Ag^+ NO_3^-	AgCl NO_3^- Na^+	NO_3^- Ag^+ AgCl
NaCl	Na^+ AgCl	NO_3^- Na^+ Cl^-	Na^+ Cl^-	Cl^- Na^+	AgCl NO_3^- Na^+	Cl^- Na^+	Cl^- Na^+ AgCl
AgCl	Ag^+ AgCl	NO_3^- AgCl	Na^+ AgCl	Cl^- AgCl	NO_3^- Ag^+ AgCl	Cl^- Na^+ AgCl	AgCl

Case 4. Both salts are soluble in water but ions form insoluble solids. Let AB and CD be soluble but AD and CB be insoluble produced solids. By pouring AB and CD into water the reaction steps are as follows:



The overall reaction is



and we obtain the following hyper operation table:

Table 7: $\text{AB} + \text{CD} \rightarrow \text{AD} + \text{CB}$

\oplus	A	B	C	D	AB	CD	AD	CB
A	A	A,B	A,C	AD	A,B	C,AD	A,AD	A,CB
B	A,B	B	CB	B,D	A,B	CB,D	B,AD	B,CB
C	A,C	CB	C	C,D	CB,A	C,D	C,AD	C,CB
D	AD	B,D	C,D	D	B,AD	D,C	D,AD	D,CB
AB	A,B	A,B	A,CB	B,AD	A,B	AD,CB	B,A,AD	A,B,CB
CD	AD,C	CB,D	C,D	C,D	AD,CB	C,D	C,D,AD	C,D,CB
AD	A,AD	AD,B	AD,C	AD,D	A,B,AD	C,D,AD	AD	CB,AD
CB	A,CB	B,CB	C,CB	D,CB	A,B,CB	C,D,CB	AD,CB	CB

Theorem 2.5. Let $H := \{A, B, C, D, AB, CD, AD, CB\}$, where A and C are cations and B and D are anions and $x \oplus y$ is the chemical interaction of x and y for every $x, y \in H$. Then (H, \oplus) is an H_v -semigroup.

Proof. Suppose that $x, y, z \in H$ and consider the following cases:

Case 1. If $\{x, y, z\} \subseteq \{A, C\}$, then $x \oplus (y \oplus z) = (x \oplus y) \oplus z$ since $(\{A, C\}, \oplus)$ is isomorphic to a sub-semihypergroup $\mathfrak{S}[\{A, C\}]$.

Case 2. If $\{x, y, z\} \subseteq \{B, D\}$, then $x \oplus (y \oplus z) = (x \oplus y) \oplus z$ since $(\{B, D\}, \oplus)$ is isomorphic to a sub-semihypergroup $\mathfrak{S}[\{B, D\}]$.

Case 3. If $\{x, y, z\} \subseteq \{A, B, AB\}$, then $x \oplus (y \oplus z) = (x \oplus y) \oplus z$ since $(\{A, B, AB\}, \oplus)$ is isomorphic to a sub-semihypergroup $\mathfrak{S}[\{A, B\}]$.

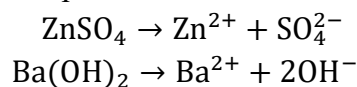
Case 4. If $\{x, y, z\} \subseteq \{C, D, CD\}$, then $x \oplus (y \oplus z) = (x \oplus y) \oplus z$ since $(\{C, D, CD\}, \oplus)$ is isomorphic to a sub-semihypergroup $\mathfrak{S}[\{C, D\}]$.

Case 5. If $AD \in \{x, y, z\}$, then $AD \in x \oplus (y \oplus z) \cap (x \oplus y) \oplus z$.

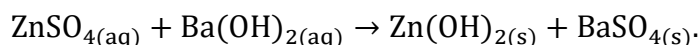
Case 6. If $CB \in \{x, y, z\}$, then $CB \in x \oplus (y \oplus z) \cap (x \oplus y) \oplus z$.

Otherwise, $\{AD, CB\} \cap (x \oplus (y \oplus z) \cap (x \oplus y) \oplus z) \neq \emptyset$. Therefore, (H, \oplus) is an H_v -semigroup.

Example 4. The zinc sulfate and barium hydroxide are both soluble salts in water. By pouring them into water, they decompose:



The cation Zn^{2+} and the anion OH^- can form an insoluble salt of Zn(OH)_2 and form a precipitate. The same reaction also takes place between Ba^{2+} and SO_4^{2-} . So, the overall reaction will result in two solids:



3. CONCLUSION AND FUTURE WORK

In this paper, we have provided some examples of hyperstructures associated with salt metathesis reactions that are examples of the phenomena when composition of two elements is a set of elements. In Theorems 2.2 and 2.3 it is shown that the considered chemical system forms a semihypergroup. In Theorems 2.4 and 2.5 we have an H_v -semigroup. Although the obtained results are interesting, more investigation on the application of this study in chemistry is required and should be considered in the future work.

Table 8: $\text{ZnSO}_4(\text{aq}) + \text{Ba}(\text{OH})_2(\text{aq}) \rightarrow \text{Zn}(\text{OH})_2(\text{s}) + \text{BaSO}_4(\text{s})$

\oplus	Zn^{+2}	SO_4^{-2}	Ba^{+2}	OH^-	ZnSO_4	$\text{Ba}(\text{OH})_2$	$\text{Zn}(\text{OH})_2$	BaSO_4
Zn^{+2}	Zn^{+2}	Zn^{+2} SO_4^{-2}	Zn^{+2} Ba^{+2}	$\text{Zn}(\text{OH})_2$	Zn^{+2} SO_4^{-2}	Ba^{+2} $\text{Zn}(\text{OH})_2$	Zn^{+2} $\text{Zn}(\text{OH})_2$	Zn^{+2} BaSO_4
SO_4^{-2}	Zn^{+2} SO_4^{-2}	SO_4^{-2}	BaSO_4	SO_4^{-2} OH^-	SO_4^{-2} Zn^{+2}	OH^- BaSO_4	SO_4^{-2} $\text{Zn}(\text{OH})_2$	SO_4^{-2} BaSO_4
Ba^{+2}	Zn^{+2} Ba^{+2}	BaSO_4	Ba^{+2}	Ba^{+2} OH^-	BaSO_4 Zn^{+2}	Ba^{+2} OH^-	Ba^{+2} $\text{Zn}(\text{OH})_2$	Ba^{+2} BaSO_4
OH^-	$\text{Zn}(\text{OH})_2$	SO_4^{-2} OH^-	Ba^{+2} OH^-	OH^-	$\text{Zn}(\text{OH})_2$	OH^- Ba^{+2}	OH^- $\text{Zn}(\text{OH})_2$	OH^- BaSO_4
ZnSO_4	Zn^{+2} SO_4^{-2}	SO_4^{-2} Zn^{+2}	BaSO_4 Zn^{+2}	$\text{Zn}(\text{OH})_2$	Zn^{+2} SO_4^{-2}	$\text{Zn}(\text{OH})_2$ BaSO_4	Zn^{+2} SO_4^{-2} $\text{Zn}(\text{OH})_2$	SO_4^{-2} Zn^{+2} BaSO_4
$\text{Ba}(\text{OH})_2$	Ba^{+2} $\text{Zn}(\text{OH})_2$	OH^- BaSO_4	Ba^{+2} OH^-	OH^- Ba^{+2}	$\text{Zn}(\text{OH})_2$ BaSO_4	Ba^{+2} OH^-	Ba^{+2} OH^- $\text{Zn}(\text{OH})_2$	Ba^{+2} OH^- BaSO_4
$\text{Zn}(\text{OH})_2$	Zn^{+2} $\text{Zn}(\text{OH})_2$	SO_4^{-2} $\text{Zn}(\text{OH})_2$	Ba^{+2} $\text{Zn}(\text{OH})_2$	OH^- $\text{Zn}(\text{OH})_2$	Zn^{+2} SO_4^{-2} $\text{Zn}(\text{OH})_2$	Ba^{+2} OH^- $\text{Zn}(\text{OH})_2$	$\text{Zn}(\text{OH})_2$	$\text{Zn}(\text{OH})_2$ BaSO_4
BaSO_4	Zn^{+2} BaSO_4	SO_4^{-2} BaSO_4	Ba^{+2} BaSO_4	OH^- BaSO_4	BaSO_4 SO_4^{-2} Zn^{+2}	Ba^{+2} OH^- BaSO_4	$\text{Zn}(\text{OH})_2$ BaSO_4	BaSO_4

Acknowledgement. The authors gratefully thank to referee for the helpful comments and recommendations leading us to improve the readability and quality of the paper.

REFERENCES

1. K. M. Chun, Chemical hyperstructures of chemical reactions for iron and idium, *J. Chungcheong Math. Soc.* **27** (2) (2014) 319–325.
2. S. -C. Chung, Chemical hyperstructures for vanadium, *J. Chungcheong Math. Soc.* **27** (2) (2014) 309–317.
3. S. -C. Chung, K. M. Chun, N. J. Kim, S. Y. Jeong, H. Sim, J. Lee and H. Maeng, Chemical hyperalgebras for three consecutive oxidation states of elements, *MATCH Commun. Math. Comput. Chem.* **72** (2) (2014) 389–402.
4. P. Corsini and V. Leoreanu, *Applications of Hyperstructures Theory*, Advanced in Mathematics, Kluwer Academic Publisher, 2003.
5. B. Davvaz, Weak algebraic hyperstructures as a model for interpretation of chemical reactions, *Iranian J. Math. Chem.* **7** (2) (2016) 267–283.
6. B. Davvaz and A. Dehgan-Nezhad, Chemical examples in hypergroups, *Ratio Mat.* **14** (2003) 71–74.

7. B. Davvaz, A brief survey of the theory of H_V -structures, Proc. 8th International Congress on Algebraic Hyperstructures and Applications, 1–9 September 2002, Samothraki, Greece, Spanidis Press, 2003, pp. 39–70.
8. B. Davvaz, A. Dehghan Nezhad and A. Benvidi, Chemical hyperalgebra: Dismutation reactions, *MATCH Commun. Math. Comput. Chem.* **67** (2012) 55–63.
9. B. Davvaz, A. Dehghan Nezhad and A. Benvidi, Chain reactions as experimental examples of ternary algebraic hyperstructures, *MATCH Commun. Math. Comput. Chem.* **65** (2) (2011) 491–499.
10. B. Davvaz and A. Dehghan Nezhad, Dismutation reactions as experimental verifications of ternary algebraic hyperstructures, *MATCH Commun. Math. Comput. Chem.* **68** (2012) 551–559.
11. B. Davvaz, A. Dehghan Nezhad and M. Mazloun-Ardakani, Chemical hyperalgebra: Redox reactions, *MATCH Commun. Math. Comput. Chem.* **71** (2014) 323–331.
12. B. Davvaz, *Semihypergroup Theory*, Elsevier/Academic Press, London, 2016.
13. F. Marty, Sur une generalization de la notion de group, In 8th Congress Math. Scandenaves, 1934, pp. 45–49.
14. Ch. E. Mortimer, *Chemistry*, Wadsworth Pub Co, 6 Sub edition, 1986.
15. T. Vougiouklis, H_V -groups defined on the same set, *Discrete Math.* **155** (1996) 259–265.
16. T. Vougiouklis, *Hyperstructures and Their Representations*, Hadronic Press, Inc, 115, Palm Harber, USA, 1994.
17. T. Vougiouklis, The fundamental relation in hyperrings. The general hyperfield, Proc. Fourth Int. Congress on Algebraic Hyperstructures and Appl. (AHA 1990), World Scientific, 1991, pp. 203–211.