Iranian J. Math. Chem. 9 (1) March (2018) 65 − 75

Iranian Journal of Mathematical Chemistry

Journal homepage: ijmc.kashanu.ac.ir

Weak Chemical Hyperstructures Associated to Electrochemical Cells

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ARTICLE INFO

Article History:

Received 8 June 2017
Accepted 5 July 2017
Published online 14 January 2018
Academic Editor: Ivan Gutman

Keywords:

H_v-semigroup
Complete part
Fundamental relation
Galvanic cell
Electrolytic cell

ABSTRACT

Algebraic hyperstructures have many applications in various sciences. The main purpose of this paper is to provide a new application of weak hyperstructures in Chemistry. More precisely, we present three different examples of hyperstructures associated to electrochemical cells. In which we prove that our hyperstructures are H_v -semigroups and we present some interesting results.

1 INTRODUCTION

Hyperstructures represent a natural extension of algebraic structures and they were introduced in 1934 by F. Marty [14]. He generalized the notion of groups by defining hypergroups. Where in a group, the operation's result of two elements is again an element while in a hypergroup, the hyperoperation's result of two elements is a non-void set. Since then, hundred of books and papers discussed and studied hyperstructures from the theoretical point of view and for their applications to many subjects of pure and applied mathematics. In [5], Corsini presented some of hyperstructures' applications to several subjects such as, geometry, fuzzy sets, automata, hypergraphs, and so on.

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DOI: 10.22052/ijmc.2017.88790.1294

The largest class of hyperstructures is the one that satisfies weak axioms, i.e., the non-empty intersection replaces the equality. These are called Hv-structures and they were introduced in 1990. The latter hyperstructures have many applications to different disciplines like Biology, Chemistry, Physics, and so on. In [1, 2], the authors analyzed the second generation phenotypes and genotypes of n-hybrid cross with a mathematical structure. They used the concepts of cyclic hypergroup and H_v-semigroup in the F_2-phenotypes and F_2-genotypes respectively with mating as a hyperoperation. Another motivation for the study of hyperstructures comes from chemical reactions. In [3, 6, 7, 8], redox, chain and dismutation reactions were provided as different examples of weak hyperstructures.

In our paper, we consider a new chemical hyperstructure using Galvanic and Electrolytic cells. And it is organized as follows: after an introduction, Section 2 presents some definitions that are used throughout the paper. Section 3 defines binary hyperstructures related to Galvanic cells, Electrolytic cells and proves that they are isomorphic. Moreover, it defines a binary hyperstructure related to both Galvanic and Electrolytic cells at the same time and investigate its properties.

2. WEAK HYPERSTRUCTURES

In this section, we present some definitions related to hyperstructures (see [4, 9, 10, 11, 12, 13]) that are used throughout the paper.

Let H be a non-empty set. Then, a mapping $\circ : H \times H \to P^*(H)$ is called a binary hyperoperation on H, where $P^*(H)$ is the family of all non-empty subsets of H. The couple $(H,\circ)$ is called a hypergroupoid. In the above definition, if A and B are two non-empty subsets of H and $x \in H$, then we define: $A \circ B = \bigcup_{a\in A, b\in B} a \circ b$, $x \circ A = \{x\} \circ A$ and $A \circ x = A \circ \{x\}$.

H_v-structures were introduced by T. Vougiouklis as a generalization of the well-known algebraic hyperstructures. Some axioms of classical algebraic hyperstructures are replaced by their corresponding weak axioms in H_v-structures. Most of H_v-structures are used in representation theory.

A hypergroupoid $(H,\circ)$ is called an H_v-semigroup if for all $x, y, z \in H$,

$$\big(x \circ (y \circ z)\big) \cap \big((x \circ y) \circ z\big) \neq \emptyset.$$

A subset K of an H_v-semigroup is an H_v-subsemigroup if K is an H_v-semigroup. An element $x \in H$ is called idempotent if $x^2 = x \circ x = x$ and an element $e \in H$ is called an identity of $(H,\circ)$ if $x \in x \circ e \cap e \circ x$, for all $x \in H$. The latter is called strong identity if $e \circ x = x \circ e \subseteq \{e, x\}$ for all $x \in H$. A hypergroupoid $(H,\circ)$ is called a semihypergroup if for every $x, y, z \in H$, we have $x \circ (y \circ z) = (x \circ y) \circ$

z and is called a quasihypergroup if for every $x \in H$, $x \circ H = H = H \circ x$. The latter condition is called the reproduction axiom. The couple $(H,\circ)$ is called a hypergroup if it is a semihypergroup and a quasi-hypergroup. Two hypergroupoids $(H,\circ)$ and $(K,*)$ are said to be isomorphic hypergroupoids, written as $H \cong K$, if there exists a bijective function $f: H \to K$ such that $f(x \circ y) = f(x) * f(y)$ for all $x, y \in H$.

3. MAIN RESULTS

This section is divided into three subsection as Galvanic Cells , Electrolytic Cells and Galvanic/Electrolytic Cells. Each section will be separately discussed in what follows:

3.1. GALVANIC CELLS

Chemical reactions involving the transfer of electrons from one reactant to another are called oxidation-reduction reactions or redox reactions. In a redox reaction, two half-reactions occur; one reactant (with less electronegativity) gives up electrons (undergoes oxidation) and another reactant (with higher electronegativity) gains electrons (undergoes reduction). For example, a piece of zinc going into a solution as zinc ions, with each *Zn* atom giving up 2 electrons, is an example of an oxidation half-reaction.

$$Zn \to Zn^{2+} + 2e^-.$$

In contrast, the reverse reaction, in which Zn^{2+} ions gain 2 electrons to become *Zn* atoms, is an example of a reduction half-reaction.

$$Zn^{2+} + 2e^- \to Zn.$$

A redox reaction result when an oxidation and reduction half-reaction are combined to complete a transfer of electrons as in the following example:

$$Zn + Cu^{2+} \to Zn^{2+} + Cu.$$

The electrons are not shown in the above redox reaction because they are neither reactants nor products but have simply been transferred from one species to another (from *Zn* to Cu^{2+} in this case). In this redox reaction, the *Zn* is referred to as the reducing agent because it causes the Cu^{2+} to be reduced to *Cu*. The Cu^{2+} is called the oxidizing agent because it causes the *Zn* to be oxidized to Zn^{2+}.

A Galvanic cell or voltaic cell is a device in which a redox reaction spontaneously occurs and produces an electric current. In order for the transfer of electrons in a redox reaction to produce an electric current and be useful, the electrons are made to pass through an external electrically conducting wire instead of being directly transferred between the oxidizing and reducing agents. The design of a Galvanic cell allows this to occur. In a Galvanic cell, two solutions, one containing the ions of the oxidation half-reaction and the other containing the ions of the reduction half-reaction, are placed in separated compartments called half-cells. For each half-cell, the metal, which is called an electrode, is placed in the solution and connected to an external wire. The electrode at which oxidation occurs is called the anode (Zn in the above example) and the electrode at which reduction occurs is called the cathode (Cu in the above example). The two half-cells are connected by a salt-bridge that allows a "current" of ions from one half-cell to the other to complete the circuit of electron current in the external wires. When the two electrodes are connected to an electric load (such as a light bulb or voltmeter) the circuit is completed, the oxidation-reduction reaction occurs, and electrons move from the anode (-) to the cathode (+), producing an electric current.

Galvanic cell consists of two half-cells, such that the electrode of one half-cell is composed of metal A (with larger electronegativity) and the electrode of the other half-cell is composed of metal B (with smaller electronegativity). The redox reactions for the two separate half-cells are given as follows:

$$A^{n+} + ne^- \to A,$$
$$B \to B^{m+} + me^-.$$

The two metals A and B can react with each other according to the following balanced equation:

$$nB + mA^{n+} \to mA + nB^{m+}.$$

Having the element Cu with greater electronegativity than that of Zn, we get that $Zn + Cu^{2+} \to Zn^{2+} + Cu$ is an example of a redox reaction occurring in a Galvanic cell. For more details about Galvanic cells, see [16].

Next, we present a commutative hyperstructure related to Galvanic cell and investigate its properties. We consider the set $H = \{A, B, A^{n+}, B^{m+}\}$ and we define a hyperoperation $\oplus_1$ on H as follows: $x \oplus_1 y$ is the result of a possible reaction between x and y in a Galvanic cell. If x and y do not react in a Galvanic cell then we set $x \oplus_1 y = \{x, y\}$. All possible spontaneous redox reactions of $\{A, B, A^{n+}, B^{m+}\}$ in a Galvanic cell are summarized in the following commutative table:

$\oplus_1$	A	B	A^{n+}	B^{m+}
A	A	$\{A,B\}$	$\{A, A^{n+}\}$	$\{A, B^{m+}\}$
B	$\{A, B\}$	B	$\{A, B^{m+}\}$	$\{B, B^{m+}\}$
A^{n+}	$\{A, A^{n+}\}$	$\{A, B^{m+}\}$	A^{n+}	$\{A^{n+}, B^{m+}\}$
B^{m+}	$\{A, B^{m+}\}$	$\{B, B^{m+}\}$	$\{A^{n+}, B^{m+}\}$	B^{m+}

In above table, if we change the names from A, B, A^{n+}, B^{m+} to a, b, c, d respectively, then the following theorem holds.

Theorem 1. Let $H=\{a, b, c, d\}$, $\oplus_1$ be the hyperoperation on H and consider the following table corresponding to $(H,\oplus_1)$:

$\oplus_1$	a	b	c	d
a	a	$\{a, b\}$	$\{a, c\}$	$\{a, \mathrm{d}\}$
b	$\{a, b\}$	b	$\{a, d\}$	$\{b, d\}$
c	$\{a, c\}$	$\{a, d\}$	c	$\{c, \mathrm{d}\}$
d	$\{a, d\}$	$\{b, d\}$	$\{c, d\}$	d

Then $(H,\oplus_1)$ is a commutative H_v-semigroup.

Proof. It is clear from the above table that $(H,\oplus_1)$ is a commutative hypergroupoid. We need to show that $(H,\oplus_1)$ is a weak associative hypergroupoid, i.e, $x \oplus_1 (y \oplus_1 z) \cap (x \oplus_1 y) \oplus_1 z \neq \emptyset$ for all $(x, y, z) \in H^3$. We have three cases for x; $x = a \text{ or } d$, $x = b$ and $x = c$:

- Case $x = a \text{ or } d$. We have that $x \in x \oplus_1 (y \oplus_1 z) \cap (x \oplus_1 y) \oplus_1 z \neq \emptyset$.
- Case $x = b$. We have that $b \oplus_1 (c \oplus_1 c) = b \oplus_1 c = \{a, d\}$ and that $(b \oplus_1 c) \oplus_1 c = \{a, d\} \oplus_1 c = \{a, c, d\}$. Thus,
$$b \oplus_1 (c \oplus_1 c)(b \oplus_1 c) \oplus_1 c \neq \emptyset.$$
Moreover, one can easily check that $b \oplus_1 (c \oplus_1 z) \cap (b \oplus_1 c) \oplus_1 z \neq \emptyset$, and that $b \oplus_1 (y \oplus_1 c) \cap (b \oplus_1 y) \oplus_1 c \neq \emptyset$. If $y \neq c$ and $z = c$ then $b \in b \oplus_1 (y \oplus_1 z) \cap (b \oplus_1 y) \oplus_1 z$.
- Case $x = c$. This case is similar to that of Case $x = b$.

Remark 1. Since $a \oplus_1 (b \oplus_1 c) = \{a, d\} \neq (a \oplus_1 b) \oplus_1 c = \{a, c, d\}$, it follows that $(H,\oplus_1)$ is not a semihypergroup.

Remark 2. $(H,\oplus_1)$ admits two identities; a and d. Moreover, a and d are strong identities.

3.2. Electrolytic Cells

Voltaic cells are driven by a spontaneous chemical reaction that produces an electric current through an outside circuit. These cells are important because they are the basis for the batteries that fuel modern society. But they aren't the only kind of electrochemical cells. The reverse reaction in each case is non-spontaneous and requires electrical energy to occur. It is possible to construct a cell that does work on a chemical system by driving an electric current through the system. These cells are called electrolytic cells (or reverse Galvanic cells), and operate through electrolysis.

Electrolysis is used to drive an oxidation-reduction reaction in a direction in which it does not occur spontaneously by driving an electric current through the system while doing work on the chemical system itself, and therefore is non-spontaneous. Electrolytic cells, like Galvanic cells, are composed of two half-cells; one is a reduction half-cell, the other is an oxidation half-cell. The direction of electron flow in electrolytic cells, however, may be reversed from the direction of spontaneous electron flow in Galvanic cells, but the definition of both cathode and anode remain the same, where reduction takes place at the cathode and oxidation occurs at the anode. Because the directions of both half-reactions have been reversed, the sign, but not the magnitude, of the cell potential has been reversed.

Electrolytic cells consist of two half-cells, such that the electrode of one half-cell is composed of metal A (with larger electronegativity) and the electrode of the other half-cell is composed of metal B (with smaller electronegativity). The redox reactions for the two separate half-cells are given as follows:

$$A \to A^{n+} + ne^{-},$$
$$B^{m+} + me^{-} \to B.$$

The two metals A and B can react with each other according to the following balanced equation:

$$mA + nB^{m+} \to nB + mA^{n+}.$$

An example of a reaction in an Electrolytic cell is:

$$Cu + Zn^{2+} \to Zn + Cu^{2+}$$

which is the reverse of the reaction described before. For more details about Electrolytic cells, see [16].

Next we present a hyperstructure related to Electrolytic cells and investigate its properties. We consider the set $H = \{A, B, A^{n+}, B^{m+}\}$ and we define a hyperoperation $\oplus_2$ on H as follows: $x \oplus_2 y$ is the result of a possible reaction between x and y in an Electrolytic cell. If x and y do not react in an electrolytic cell then we set $x \oplus_2 y = \{x, y\}$.

All possible spontaneous redox reactions of $\{A, B, A^{n+}, B^{m+}\}$ in an electrolytic cell are summarized in the following commutative table:

$\oplus_2$	A	B	A^{n+}	B^{m+}
A	A	$\{A, B\}$	$\{A, A^{n+}\}$	$\{A^{n+}, B\}$
B	$\{A, B\}$	B	$\{A^{n+}, B\}$	$\{B, B^{m+}\}$
A^{n+}	$\{A, A^{n+}\}$	$\{A^{n+}, B\}$	A^{n+}	$\{A^{n+}, B^{m+}\}$
B^{m+}	$\{A^{n+}, B\}$	$\{B, B^{m+}\}$	$\{A^{n+}, B^{m+}\}$	B^{m+}

In the above table, if we change the names from A, B, A^{n+}, B^{m+} to a, b, c, d respectively, then the following theorem holds.

Theorem 2. Let $H=\{a, b, c, d\}$, $\oplus_2$ be the hyperoperation on H and consider the following table corresponding to $(H,\oplus_2)$:

$\oplus_2$	a	b	c	d
a	a	$\{a, b\}$	$\{a, c\}$	$\{b, c\}$
b	$\{a, b\}$	b	$\{b, c\}$	$\{b, \mathrm{d}\}$
c	$\{a, c\}$	$\{b, c\}$	c	$\{c, d\}$
d	$\{b, c\}$	$\{b, d\}$	$\{c, d\}$	d

Then $(H,\oplus_2)$ is a commutative H_v-semigroup.

Proof. Let $f: (H,\oplus_1) \to (H,\oplus_2)$ defined as follows:

$$f(a) = b, f(b) = a, f(c) = d \text{ and } f(d) = c.$$

It is easy to see that f is an isomorphism and thus, $(H,\oplus_1) \cong (H,\oplus_2)$. The latter and Theorem 1 imply that $(H,\oplus_2)$ is a commutative H_v-semigroup.

Remark 3. $(H,\oplus_2)$ admits two identities; b and c. Moreover, b and c are strong identities.

3.3. Galvanic/Electrolytic Cells

We present a commutative hyperstructure related to Galvanic/Electrolytic cells and investigate its properties. We consider the set $H=\{A, B, A^{n+}, B^{m+}\}$ and we define a hyperoperation $\oplus$ on H as follows: $x \oplus y$ is the result of a possible reaction between x and y in either a Galvanic cell or in an Electrolytic cell. If x and y neither react in a Galvanic cell nor in an Electrolytic cell then we set $x \oplus y = \{x, y\}$.

All possible spontaneous redox reactions of $\{A, B, A^{n+}, B^{m+}\}$ in a Galvanic/Electrolytic cell are summarized in the following commutative table:

$\oplus$	A	B	A^{n+}	B^{m+}
A	A	$\{A, B\}$	$\{A, A^{n+}\}$	$\{A^{n+}, B\}$
B	$\{A, B\}$	B	$\{A, B^{m+}\}$	$\{B, B^{m+}\}$
A^{n+}	$\{A, A^{n+}\}$	$\{A, B^{m+}\}$	A^{n+}	$\{A^{n+}, B^{m+}\}$
B^{m+}	$\{A^{n+}, B\}$	$\{B, B^{m+}\}$	$\{A^{n+}, B^{m+}\}$	B^{m+}

Remark 4. We can define $(H,\oplus)$ as follows:

$$x \oplus y = \begin{cases} x \oplus_1 y & \text{if } x \oplus_2 y = \{x,y\}; \\ x \oplus_2 y & \text{if } x \oplus_1 y = \{x,y\}; \\ \{x,y\} & \text{if } x \oplus_1 y = x \oplus_2 y. \end{cases}$$

In the above table, if we change the names from *A, B,* A^{n+}, B^{m+} to *a, b, c, d* respectively, then the following theorem holds.

Theorem 3. Let $H=\{a, b, c, d\}$, $\oplus$ be the hyperoperation on H and consider the following table corresponding to $(H,\oplus)$:

$\oplus$	a	b	c	d
a	a	$\{a, b\}$	$\{a, c\}$	$\{b, c\}$
b	$\{a, b\}$	b	$\{a, d\}$	$\{b, d\}$
c	$\{a, c\}$	$\{a, d\}$	c	$\{c, d\}$
d	$\{b, c\}$	$\{b, d\}$	$\{c, d\}$	d

Then $(H,\oplus)$ is a commutative H_v-semigroup.

Proof. It is clear from the above table that $(H,\oplus)$ is a commutative hypergroupoid. We need to show that $(H,\oplus)$is a weak associative hypergroupoid. Let $(x,y,z) \in H^3$. We have four cases for *x; x=a, x=b, x=c* and *x=d*:

- Case $x = a$. We have that $a \oplus (d \oplus d) = a \oplus d = \{b,c\}$ and that $(a \oplus d) \oplus d = \{b,c\} \oplus d = \{b,c,d\}$. Thus, $a \oplus (d \oplus d) \cap (a \oplus d) \oplus d \neq \emptyset$. Moreover, one can easily check that $a \oplus (d \oplus z) \cap (a \oplus d) \oplus z$ and that $a \oplus (y \oplus d) \cap (a \oplus y) \oplus d \neq \emptyset$. If $y \neq d$ and $z \neq d$ then $a \in a \oplus (y \oplus z) \cap (a \oplus y) \oplus z$.
- Case $x = b$. We have that $b \oplus (c \oplus c) = b \oplus c = \{a,d\}$ and that $(b \oplus c) \oplus c = \{a,d\} \oplus c = \{a,c,d\}$. Thus, $b \oplus (c \oplus c) \cap (b \oplus c) \oplus c \neq \emptyset$. Moreover, one can easily check that $b \in b \oplus (c \oplus z) \cap (b \oplus c) \oplus z \neq \emptyset$ and that $b \oplus (y \oplus c) \cap (a \oplus y) \oplus c \neq \emptyset$. If $y \neq c$ and $z \neq c$ then $b \in b \oplus (y \oplus z) \cap (b \oplus y) \oplus z$.
- Case $x = c$. We have that $c \oplus (b \oplus b) = c \oplus b = \{a,d\}$ and that $(c \oplus b) \oplus b = \{a,d\} \oplus b = \{a,b,d\}$. Thus, $c \oplus (b \oplus b) \cap (c \oplus b) \oplus b \neq \emptyset$. Moreover, one can easily check that $c \oplus (b \oplus z) \cap (c \oplus b) \oplus z \neq \emptyset$ and that $c \oplus (y \oplus b) \cap (c \oplus y) \oplus b \neq \emptyset$. If $y \neq b$ and $z \neq b$ then $c \in c \oplus (y \oplus z) \cap (c \oplus y) \oplus z$.
- Case $x = d$. We have that $d \oplus (a \oplus a) = d \oplus a = \{b,c\}$ and that $(d \oplus a) \oplus a = \{b,c\} \oplus a = \{a,b,c\}$. Thus, $d \oplus (a \oplus a) \cap (d \oplus a) \oplus$

$a \neq \emptyset$. Moreover, one can easily check that $d \oplus (a \oplus z) \cap (d \oplus a) \oplus z \neq \emptyset$ and that $d \oplus (y \oplus a) \cap (d \oplus y) \oplus a \neq \emptyset$. If $y \neq a$ and $z \neq a$ then $d \in d \oplus (y \oplus z) \cap (d \oplus y) \oplus z$.

Remark 5. Every element in $(H,\oplus)$ is idempotent. This is trivial from chemical point of view as no reaction exists in an electrochemical cell between two identical elements, so, the element is unchanged.

Proposition 4. $(H,\oplus)$ is not a quasi-hypergroup nor a semihyperegroup.

Proof. Since d is not an element in $a \oplus H$, it follows that $(H,\oplus)$ is not a quasi-hypergroup. Having $a \oplus (d \oplus d) = \{b,c\} \neq (a \oplus d) \oplus d = \{b,c\} \oplus d = \{b,c,d\}$ implies that $(H,\oplus)$ is not a semihypergroup.

Proposition 5. $(H,\oplus)$ does not admit an identity element.

Proof. Since a,b,c,d are not elements of $a \oplus d, b \oplus c, c \oplus b, d \oplus a$, it follows that none of our elements is an identity.

Remark 6. Proposition 5 implies that there exists no element x in H (in a Galvanic/Electrolytic cell) such that the following reaction occurs for all y in H and some z in H:

$$x + y \to y + z.$$

Remark 7. Remark 2, Theorem 2 and Proposition 5 imply that $(H,\oplus)$ is not isomorphic to $(H,\oplus_1)$ nor to $(H,\oplus_2)$.

Proposition 6. There are only two H_v-subsemigroups of $(H,\oplus)$ up to isomorphism.

Proof. It is easy to see that $(\{a\},\oplus)$ and $(\{a,b\},\oplus)$ are the only H_v-subsemigroups of $(H,\oplus)$ up to isomorphism. Moreover, $(\{a\},\oplus)$ and $(\{a,b\},\oplus)$ are hypergroups.

Definition 7. Let $(H,\circ)$ be an H_v-semigroup and A be a non-empty subset of H. A is a complete part of H if for any natural number n and for all hyperproducts $P \in H_H(n)$, the following implication holds:

$$A \cap P \neq \emptyset \to P \subseteq A.$$

Proposition 8. $(H,\oplus)$ has no proper complete parts.

Proof. Let $A \neq \emptyset$ be a complete part of $(H,\oplus)$. We consider the following cases for A:

- Case $a \in A$. Having $a \in a \oplus x$, $x \in a \oplus x$ for all $x \in \{a, b, c\}$ imply that $x \in a \oplus x \subseteq A$. We get now that $b \in A$. Since $b \in b \oplus d$ and $d \in b \oplus d$, it follows that $d \in b \oplus d \subseteq A$. Thus, $A = H$.
- Case $b \in A$. Having $b \in b \oplus a$ implies that $a \in b \oplus a \subseteq A$. The latter implies that $a \in A$ and thus $A = H$ by the first case.
- Case $c \in A$. Having $c \in c \oplus a$ implies that $a \in c \oplus a \subseteq A$. The latter implies that $a \in A$ and thus $A = H$ by the first case.
- Case $d \in A$. Having $d \in c \oplus d$ implies that $c \in c \oplus d \subseteq A$. The latter implies that $c \in A$ and thus $A = H$ by the previous case.

Therefore, $(H,\oplus)$ has no proper complete part.

The main tools connecting the class of hyperstructures with the classical algebraic structures are the fundamental relations. The fundamental relation has an important role in the study of semihypergroups and especially of hypergroups.

Definition 9. For all $n > 1$, we define the relation β_n on an H_v-semigroup $(H,\circ)$ as follows:

$$x\beta_n\, y \text{ if there exist } a_1, \cdots, a_n \in H \text{ such that } \{x, y\} \subseteq \prod_{i=1}^{n} a_i.$$

and we set $\beta = \bigcup_{n\geq 1} \beta_n$, where $\beta_1 = \{(x, x)|\ x \in H\}$ is the diagonal relation on H.

This relation was introduced by Koskas [13] and studied mainly by Corsini [4], Davvaz [9], Davvaz and Leoreanu-Fotea [11], Freni [12], Vougiouklis [15], and many others. Clearly, the relation β is reflexive and symmetric. Denote by β^* the transitive closure of β.

The β^* is called the *fundamental equivalence relation* on H and it is the smallest strongly regular relation on H. If H is a hypergroup then $\beta = \beta^*$ [12] and H/β^* is called the *fundamental group*.

Proposition 10. $(H,\oplus)$ has a trivial fundamental group.

Proof. Since $\{a, b\} \subseteq a \oplus b$, it follows that $a\beta_2\, b$. Similarly, we obtain $a\beta_2$ c, $b\beta_2 d$, $c\beta_2 d$. Having β^* the transitive closure of β, one can easily see that $x\beta^* y$ for all $(x, y) \in H^2$. Thus, $|H/\beta^*| = 1$.

4. CONCLUSION

This paper provided a new chemical hyperstructure on electrochemical cells that is not equivalent to any of the studied chemical hyperstructures before.

ACKNOWLEDGEMENT. We appreciate the assistance of Dr. Sami Tlais in understanding some concepts and notions related to both Galvanic and Electrolytic cells.

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