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On Common Neighborhood Graphs II

Asma Hamzeh[1], **Ali Iranmanesh**[1,•], **Samaneh Hossein–Zadeh**[1], **Mohammad Ali Hossein–Zadeh**[1] **and Ivan Gutman**[2]

[1]Department of Mathematics, Faculty of Mathematical Sciences,Tarbiat Modares University, P. O. Box 14115-137, Tehran, Iran
[2]Faculty of Science, University of Kragujevac, P. O. Box 60, 34000 Kragujevac, Serbia

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ABSTRACT

Let G be a simple graph with vertex set $V(G)$. The common neighborhood graph or congraph of G, denoted by $con(G)$, is a graph with vertex set $V(G)$, in which two vertices are adjacent if and only if they have at least one common neighbor in G. We compute the congraphs of some composite graphs. Using these results, the congraphs of several special graphs are determined.

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1 Introduction and Preliminaries

Let G be a simple graph with vertex set $V(G)$ and edge set $E(G)$. For any vertex $v \in V(G)$, the set of neighbors of v is the set $N_v(G) = \{u \in V(G) | uv \in E(G)\}$. We say that $v \in V(G)$ is an isolated vertex if $N_G(v)$ is an empty set. The distance between the vertices u and v of G denoted by $d_G(u,v)$. ($d(u,v)$for short), is defined as the length of the shortest path connecting u and v.

The complement of a graph G is a graph H on the same vertices such that two vertices of H are adjacent if and only if they are not adjacent in G. The graph H is usually denoted by $\bar{G}$.The minimum length of a cycle in a graph G is called the girth of G. We

•Corresponding author (Email: iranmanesh@modares.ac.ir)
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now define several kinds of products of pairs of graphs; see [14] for details.The union of the simple graphs G and H is the graph $G \cup H$ with vertex set $V(G) \cup V(H)$ and edge set $E(G) \cup E(H)$. If G and H are disjoint, then we refer to their union as a disjoint union. Suppose that G and H are two graphs with disjoint vertex sets. Their Cartesian product $G \times H$ is a graph such that $V(G \times H) = V(G) \times V(H)$, and two vertices (u_1, v_1) and (u_2, v_2) are adjacent in $G \times H$ if and only if either $u_1 = u_2$ and v_1 is adjacent with v_2, or $v_1 = v_2$ and u_1 is adjacent with u_2. The join $G + H$ of the graphs G and H is the graph union $G \cup H$ together with all the edges joining $V(G)$ and $V(H)$. The tensor product $G \otimes H$ of the graphs G and H is the graph with vertex set $V(G) \times V(H)$ in which (u_1, v_1) is adjacent with (u_2, v_2) whenever $u_1 u_2 \in E(G)$ and $v_1 v_2 \in E(H)$. The strong product $G \Omega H$ of G and H has the vertex set $V(G \Omega H) = V(G) \times V(H)$ and two distinct vertices (u_1, v_1) and (u_2, v_2) of $G \Omega H$ are adjacent if $u_1 = u_2$ and $v_1 v_2 \in E(G)$., or $u_1 u_2 \in E(G)$ and $v_1 = v_2$, or $u_1 u_2 \in E(G)$ and $v_1 v_2 \in E(H)$.. For given vertices $y \in V(G)$ and $z \in V(H)$, a splice of G and H by vertices y and z, $(G.H)(y, z)$, is defined by identifying the vertices y and z in the union of G and H [10]. Hou and Shiu [13] introduced an edge version of corona product as follows.

Let G and H be two graphs on disjoint sets of n_1, n_2 vertices and m_1, m_2 edges, respectively. The edge corona $G \lozenge H$ is defined as the graph obtained by taking one copy of G and m_1 copies of H, then joining two end-vertices of the i-th edge of G to every vertex in the i-th copy of H.

Now, we define the Hajós join which is introduced in [11]. Let G and H be two graphs, $vw \in E(G)$, and $xy \in E(H)$. Then the Hajós join of these two graphs, which is denoted by $G \Delta H$,is a new graph that combines the two graphs by identifying vertices v and x into a single vertex, removing the two edges vw and xy, and adding a new edge wy. For example, if G and H are cycles of length p and q respectively, then the Hajós join of these two cycles is itself a cycle, of length $p + q - 1$.

Let G be a simple graph with vertex set $\{v_1, v_2, \dots, v_n\}$. The common neighborhood graph (congraph) of G, denoted by $con(G)$, is a graph with the vertex set $\{v_1, v_2, \dots, v_n\}$. in which two vertices are adjacent if and only if they have at least one common neighbor in the graph G [1, 2].

Congraphs have been investigated in several earlier works [1, 2, 6, 12, 15]. In [12], we obtained some results on congraphs of graph products. In this paper we continue this study and report additional results along these lines.

It should be noted that in two earlier works [3, 4] the so-called derived graph $G^\dagger$ of the graph G was considered. The derived graph $G^\dagger$ has the same vertex set as the parent graph G, and two vertices of $G^\dagger$ are adjacent if and only if their distance in G is equal to

two. It is immediately seen that $G^{\dagger} = con(G)$ if and only if the parent graph G does not contain triangles. Thus, in particular, $G^{\dagger} = con(G)$ holds whenever G is bipartite.
The notations used in this paper is standard and taken mainly from [5, 14]. In what follows, the graphs considered are assumed to be simple. If a graph has parallel edges, we consider these as a single edge.

2 Common Neighborhood Graphs Of Some Graph Operations

In this section we obtain $con(G)$ for some operations on two graphs. We begin with the tensor product. To do this, we state the following lemma which immediately follows from the definition of the operation $\otimes$.

Lemma 2.1. Let (v_i, u_j) and (v_r, u_s) be two vertices of $G\otimes H$. Then $(v_k, u_t) \in N_{G\otimes H}(v_i, u_j) \cap N_{G\otimes H}(v_r, u_s)$ if and only if $v_k \in N_G(v_i) \cap N_G(v_r)$ and $u_t \in N_H(u_j) \cap N_H(u_s)$.

Theorem 2.2. Let G and H be two graphs without isolated vertices. Then

$$con(G \otimes H) = con(G) \Omega \, con(H).$$

Proof. Let (v_i, u_j) and (v_r, u_s) be two vertices of $G\otimes H$ such that $v_i \neq v_r$ and $u_j \neq u_s$. If $(v_i, u_j)(v_r, u_s)$ is an edge of $con(G\otimes H)$, then there is a vertex $(v_k, u_t) \in V(G\otimes H)$ such that $(v_k, u_t) \in N_{G\otimes H}(v_i, u_j) \cap N_{G\otimes H}(v_r, u_s$So by Lemma 2.1, $v_k \in N_G(v_i) \cap N_G(v_r)$ and $u_t \in N_H(u_j) \cap N_H(u_s)$. Therefore $v_i v_r \in E(con(G))$ and $u_j u_s \in E(con(H))$. This means that for $v_i \neq v_r$ and $u_j \neq u_s$it holds that $(v_i, u_j)(v_r, u_s)$ is an edge of $con(G\otimes H)$ if and only if $v_i v_r \in E(con(G))$ and $u_j u_s \in E(con(H))$..

Assume that $v_i = v_r = v$. If $(v, u_j)(v, u_s)$ is an edge of $con(G\otimes H)$, then there is a vertex (v_k, u_t) such that $(v_k, u_t) \in N_{G\otimes H}(v, u_j) \cap N_{G\otimes H}(v, u_s)$. By Lemma 2.1, we have $v_k \in N_G(v)$ and $u_t \in N_H(u_j) \cap N_H(u_s)$. So if $v_i = v_r$, then $u_j u_s \in E(con(H))$. Therefore, for $v_i = v_r$ it holds that $(v_i, u_j)(v_r, u_s)$ is an edge of $con(G\otimes H)$ if and only if $v_i = v_r$ and $u_j u_s \in E(con(H))$. Similarly if $u_j = u_s$, then $(v_i, u_j)(v_r, u_s)$ is an edge of $con(G\otimes H)$ if and only if $u_j = u_s$ and $v_i v_r \in E(con(G))$.

Hence $con(G \otimes H) = (con(G) \otimes con(H)) \cup (con(G) \times con(H)) = con(G) \, \Omega \, con(H)$ and this completes the proof. ■

In the following theorem, we determine the congraph of Hajós join.

Theorem 2.3. Let G and H be two graphs with the girth at least 5. Then

$$\begin{aligned} con(G\Delta H) = {} & (con(G)\cdot con(H))(v,x) - E(y+N_H(x)) - E(w+N_G(v)) - E(x+N_H(y)) \\ & - E(v+N_G(w)) \cup E((N_G(v)-\{w\})+(N_H(x)-\{y\})) \cup E(N_G(w)+y) \\ & \cup E(N_H(y)+w), \end{aligned}$$

where for two vertices r, s, the notation $E(r + N(s))$ denotes the edges of the join of the vertex r and the neighbors of s.

Proof. In the structure of Hajós join, if we don't remove two edges vw and xy, and don't add a new edge wy, we can arrive to splice of two graphs G and H. So we consider the graph $\big(con(G).con(H)\big)(v,x)$ as the base of the common neighborhood graph of the Hajós join of G and H. Then we investigate the effect of removing the two edges vw and xy, and adding a new edge wy.

Since the girth of the graph G is at least 5, when we remove the edge vw, all the edges wr and vs in $con(G)$, $r \in N_G(v)$ and $s \in N_G(w)$, that have v and w as the common neighbor, respectively, will be deleted. Similarly when we eliminate the edge xy, all the edges ya and xb in $con(H)$, $a \in N_H(x)$ and $b \in N_H(y)$, that have x and y as the common neighbor, respectively, will be deleted. Continuing this argument, when we identify the vertices v and x into a single vertex, then $N_G(v) - w$ and $N_H(x) - y$ will have a common neighbor. So each vertex in $N_G(v) - w$ will be adjacent to each vertex in $N_H(x) - y$. By adding the new edge wy, w will become the common neighbor between y and $N_G(w)$ and y the common neighbor between w and $N_H(y)$. ■

Applying the Hajós join to two copies of K_4 by identifying a vertex from each copy into a single vertex, deleting an edge incident to the combined vertex within each subgraph, and adding a new edge connecting the endpoints of the deleted edges, produces the Moser spindle, see Fig. 1. As an application we characterize the common neighborhood graph of the Moser spindle.

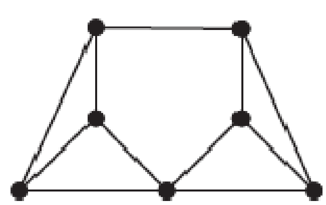

Figure 1. The Moser Spindle Graph.

Corollary 2.4. The common neighborhood graph of the Moser spindle is $K_7 - e$.

In the next theorem, we compute the common neighborhood graph of the edge corona product of graphs. One can see that the edge corona product of G with a complete graph K_t, and the common neighborhood graph of G are subgraphs of $G \lozenge H$.

Theorem 2.5. Let G be a graph with n vertices and m edges and H be a graph with t vertices. Then

$$con(G\lozenge H) = (G\lozenge K_t) \cup con(G) \cup (\bigcup_{e_k=v_iv_j} \left(N_G(v_i) \cup N_G(v_j)\right) + H_k)$$
$$\cup (\bigcup_{e_i=v_lv_q, e_j=v_lv_{q'}} (H_i + H_j)).$$

Proof. Let $V = \{v_1, \dots, v_n\}$, $E(G) = \{e_1, \dots, e_m\}$, and $V(H) = \{u_1, \dots, u_t\}$. Denote the i-th copy of H in $G \lozenge H$, by H_i. Each two vertices of H_i have the end vertices of e_i as common neighbors, So the induced subgraph of $con(G \lozenge H)$ on each H_i is a complete graph. On the other hand, a vertex in H_i has a common neighbor with a vertex in H_j if and only if the edges e_i and e_j are adjacent. So the induced subgraph of $G \lozenge H$ on the vertices $H_i \cup H_j$ is $H_i + H_j$ if and only if e_i and e_j are adjacent in G and there is no edge between H_i and H_j if e_i and e_j are not adjacent in G.

We now consider the vertices $\{v_1, \dots, v_n\}$. Clearly, v_i and v_j have a common neighbor v_k in $G \lozenge H$ if and only if v_k is their common neighbor in G. Also v_i and v_j have a common neighbor u_r in $G \lozenge H$, if and only if v_iv_j is an edge of G.

Finally, a vertex v_s in G has a common neighbor with a vertex u_r in H_k if and only if v_s is in $N_G(v_i) \cup N_G(v_j)$, where $e_k = v_iv_j$. This completes the proof. ■

By definition, the edge corona $T \lozenge S_n$ of a tree T of order n and S_n is the graph obtained by taking one copy of T and $n-1$ copies of S_n and then joining two end-vertices of the i-th edge of T to every vertex in the i-th copy of S_n.

Corollary 2.6. The common neighborhood graph of the edge corona product of graphs K_2 and S_n satisfies $con(K_2 \lozenge S_n) = K_{n+2}$.

3. RELATION BETWEEN SOME SPECIAL GRAPHS AND THEIR CONGRAPHS

In this section we compute the common neighborhood graphs of the central graph, line graph, shadow graph, and Mycielski graph.So we should first define these graphs.

For a given graph G, the line graph of G is denoted by $L(G)$ and the vertices of $L(G)$ are the edges of G. Two edges of G that share a vertex are considered to be adjacent in $L(G)$. The subdivision graph of the graph G is denoted by $S(G)$ and is the graph obtained by inserting an additional vertex in each edge of G. Equivalently, each edge of G is replaced by a path of length 2.

For a given graph G, we do an operation on G by subdividing each edge exactly once and joining each pair of vertices of the original graph which were previously non-adjacent. The graph obtained by this process is said to be the central graph of G, denoted by $C(G)$, [17, 18, 19].

The shadow graph $D_2(G)$ of a connected graph G is constructed by taking two copies of G say G' and G'' and joining each vertex u' in G' to the neighbors of the corresponding vertex u'' in G''. For example, $D_2(C_4)$ is depicted in Fig. 2.

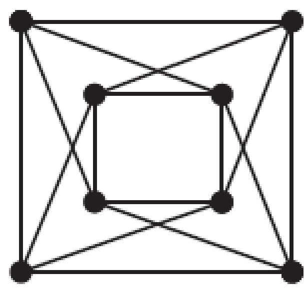

Figure 2. The Shadow Graph $D_2(C_4)$.

The Mycielski graph of G was introduced by J. Mycielski [16] for the purpose of constructing triangle–free graphs with arbitrarily large chromatic number. This graph has been much studied [7, 8, 9].

Let G be a graph with vertex set $\{v_1, v_2, \dots, v_n\}$. The Mycielski graph $\mu(G)$ of G contains G itself as an isomorphic subgraph, together with $n+1$ additional vertices: a vertex $u_i$which corresponds to each vertex v_i of G, and another vertex w. Each vertex u_i is connected by an edge to w, so that these vertices form a subgraph in the form of a star $K_{1,n}$. In addition, for each edge v_iv_j of G, the Mycielski graph includes two edges, u_iv_j and v_iu_j. In Fig. 3 we shows Mycielski's construction applied to a 5-vertex cycle. The resulting Mycielskian is the Grötzsch graph, an 11-vertex graph with 20 edges. The Grötzsch graph is the smallest triangle–free 4-chromatic graph.

Theorem 3.1. Let G be a graph. Then

$$con(C(G)) = G \cup L(G) \cup con(\overline{G}) \cup \left(\bigcup_{e=uv\in E(G)} [v_e + (V(G) - (N_G(u) \cup N_G(v)))]\right).$$

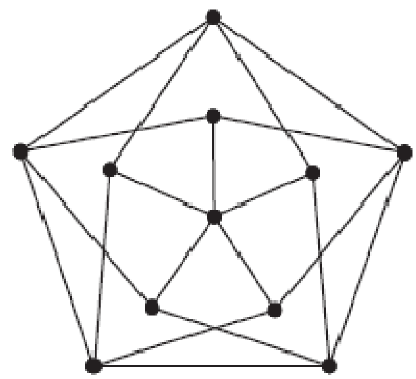

Figure 3. The Grötzsch Graph.

Proof. Let $V(G) = \{v_1, \dots, v_n\}$ and $E(G) = \{e_1, \dots, e_m\}$. So the set of vertices of $C(G)$ is $V(C(G)) = \{v_1, \dots, v_n, v_{e_1}, \dots, V_{e_m}\}$, where v_{e_i} is the vertex inserted in the edge e_i, $(1 \le i \le m)$. We determine the graph $con(C(G))$ in three steps:

(i) We find the edges between the vertices of $\{v_1, v_2 \dots, v_n\}$. The vertices v_i and v_j have a common neighbor in the set $\{v_{e_1}, \dots, V_{e_m}\}$, of graph $C(G)$ if and only if $v_i v_j$ is an edge in the graph G. Also the vertices v_i and v_j have a common neighbor in the set $\{v_1, \dots, v_n\}$ of the graph $C(G)$ if and only if v_i and v_j have a common neighbor in the graph $\bar{G}$. So the subgraph induced by the vertices $v_1, \dots, v_n$ in the graph $C(G)$ is $G \cup con(\bar{G})$.

(ii) We consider the subgraph of $C(G)$ induced by the set $\{v_{e_1}, \dots, V_{e_m}\}$,. It is easy to see that v_{e_i} and v_{e_j} do not have common neighbors in $\{v_{e_1}, \dots, V_{e_m}\}$,. On the other hand, v_{e_i} and v_{e_j} have the vertex v_t as common neighbor in $\{v_1, v_2 \dots, v_n\}$. if and only if the edges e_i and e_j have the vertex v_t as the common vertex in G. Therefore the respective induced subgraph is $L(G)$.

(iii) We find the edges between $\{v_1, v_2, \dots, v_n\}$ and $\{v_{e_1}, \dots, v_{e_m}\}$. Let $e_i = v_r v_s$ be an edge of G. So $N_{C(G)}(v_{e_i}) = \{v_r, v_s\}$ and this means that v_{e_i} is adjacent in $con(C(G))$ to the vertices that are neighbors of v_r and v_s. By the definition of $C(G)$, the edges between $\{v_1, v_2, \dots, v_n\}$ and $\{v_{e_1}, \dots, v_{e_m}\}$ are $\bigcup_{e=uv \in E(G)} [v_e + \left(V(G) - \left(N_G(u) \cup N_G(v)\right)\right)]$.

Combining (i), (ii), and (iii), the theorem follows. ■

In the graph G, let $\{e_1, \dots, e_k\}$ be all of the edges incident to vertex u. We denote the set of $\{v_{e_1}, \dots, v_{e_k}\}$ in the graph $L(G)$ by $N_G(u)$. That v_{e_i} is a vertex of $L(G)$ corresponding to an edge e_i of G.

Theorem 3.2. Let G be a graph. Then $con(L(G)) = \bigcup_{e=uv\in E(G)} (N_G^{'}(u) + N_G^{'}(v))$.

Proof. Consider the vertices $\{v_{e_1}, \dots, v_{e_m}\}$ in $L(G)$. If $e_i = v_r v_s$ is an edge in G, then v_{e_i} can be as the common neighbor of the sets $N'_G(v_r)$ and $N'_G(v_s)$. So for each $e_i = v_r v_s$ in G, $N'_G(v_r) + N'_G(v_s)$ is the subgraph of $con(L(G))$. ■

Theorem 3.3. Let G be a graph without isolated vertices and G' and G'' be two copies of G. Then $con(D_2(G)) = D_2(con(G)) \bigcup \{v_i^{'} v_i^{''} \mid v_i^{'} \in V(G'), v_i^{''} \in V(G''), 1 \le i \le |V(G)|\}$.

Proof. Suppose that $V(G) = \{v_1, v_2, \dots, v_n\}$, $V(G') = \{v'_1, v'_2, \dots, v'_n\}$, and $V(G'') = \{v''_1, v''_2, \dots, v''_n\}$,. By definition of the shadow graph, it is easy to see that $v'_t \in N_{G'}(v'_r) \cap N_{G'}(v'_s)$ if and only if $v''_t \in N_{G'}(v'_r) \cap N_{G'}(v'_s)$. Similarly, $v''_t \in N_{G'}(v''_r) \cap N_{G'}(v''_s)$ if and only if $v''_t \in N_{G'}(v'_r) \cap N_{G'}(v'_s)$. Therefore, the subgraph of $con(D_2(G))$ induced on $V(G')$ is $con(G')$ and induced on $V(G'')$ is $con(G'')$. ■

We now determine the edges between $V(G')$ and $V(G'')$. To do this, for two vertices v_i and v_j, $i \neq j$, we use the following facts resulting from the definition of shadow graph:

1) $v_k^{'} \in N_{G'}(v_i^{'}) \bigcap N_{G''}(v_j^{''})$ if and only if $v_k^{''} \in N_{G'}(v_i^{'}) \bigcap N_{G''}(v_j^{''})$.

2) $v_k^{'} \in N_{G'}(v_i^{'}) \bigcap N_{G'}(v_j^{'})$ if and only if $v_k^{'} \in N_{G'}(v_i^{'}) \cap N_{G''}(v_j^{''})$.

Therefore, uv is an edge of $con(G)$ if and only if $u'v''$ and $v'u''$ are edges of $con(D_2(G))$. On the other hand, since G has no isolated vertices, for each i, $1 \le i \le |V(G)|$, $v'_i v''_i$ are edges of $con(D_2(G))$. and the proof is completed. ■

Corollary 3.4. For path P_n and complete graph K_2 the following equality holds:

$$con(D_2(P_n)) = con(P_n) \times K_2 .$$

Theorem 3.5. Let G be a graph with n vertices. Then the congraph of its Mycielski graph contains $con(G)$ as an isomorphic subgraph, together with $n + 1$ additional vertices: a vertex u_i corresponding to each vertex v_i of $con(G)$ such that the induced graph of $\mu(G)$ on them is K_n and another vertex w. Each vertex v_i is connected by an edge to w, so that these vertices form a subgraph in the form of a star $K_{1,n}$. In addition, for each edge $v_i v_j$ of $con(G)$, the common neighborhood graph of the Mycielski graph includes two edges,$u_i v_j$ and $v_i u_j$.

Proof. Since in the Mycielski graph, each vertex u_i is connected by an edge to w, so vertex w is the common neighborhood of vertices $u_1, \dots, u_n$ in the graph $\mu(G)$ and this implies that the subgraph of $\mu(G)$ induced on these vertices is K_n . It is clear that $v_t \in N_{\mu(G)}(v_r) \cap N_{\mu(G)}(v_s)$ if and only if $u_t \in N_{\mu(G)}(v_r) \cap N_{\mu(G)}(v_s)$, so the subgraph of $\mu(G)$ induced on $\{v_1, \dots, v_n\}$ is $con(G)$. Also by the definition of Mycielski graph, since G has not isolated vertices, the vertices u_i are common neighborhoods of the vertices v_j and w. This implies that,$v_i w\, 1 \leq i \leq m$are edges of $con(\mu(G)$).

Now we obtain the edges between $\{v_1, \dots, v_n\}$ and $\{u_1, \dots, u_n\}$. Let the vertex v_k be the common neighbor of the vertices v_i and v_j. By the definition of Mycielski graph, we have the following cases:

Case 1. The vertex v_k is in the common neighborhood of vertices v_j and u_j in graph G. This implies that $v_j u_j$, $1 \leq j \leq m$are edges of the congraph.

Case 2. The vertex v_k is in the common neighborhood of the vertices v_j and u_i, v_i and u_j in graph G. This implies that for each edge $v_i v_j$ of $con(G)$, the common neighborhood graph of $\mu(G)$ includes two edges $u_i v_j$ and $v_i u_j$ and this completes the proof. ■

As an application we compute the common neighborhood graph of Grötzsch graph.

Corollary 3.6. Let $C_5 : v_1v_2v_3v_4v_5v_1$. Then the common neighborhood graph of the Grötzsch graph is determined via $(w + C_5) \bigcup K_5 \bigcup \{v_i u_{i+1} \mid 1 \leq i \leq 4\} \bigcup \{v_{i+1} u_i \mid 1 \leq i \leq 4\} \bigcup v_1 u_5 \bigcup v_5 u_1$.

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