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Weak Algebraic Hyperstructures as a Model for Interpretation of Chemical Reactions

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ABSTRACT The concept of weak algebraic hyperstructures or H_v-structures constitute a generalization of the well-known algebraic hyperstructures (semihypergroup, hypergroup and so on). The overall aim of this paper is to present an introduction to some of the results, methods and ideas about chemical examples of weak algebraic hyperstructures. In this paper after an introduction of basic definitions and results about weak algebraic hyperstructures, we review:

(1) Weak algebraic hyperstructures associated with chain reactions.
(2) Weak algebraic hyperstructures associated with dismutation reactions.
(3)Weak algebraic hyperstructures associated with redox reactions.

KEYWORDS Weak algebraic hyperstructure • hypergroup • H_v-group • chain reaction • dismutation reaction • redox reaction.

1. INTRODUCTION

The hyperstructure notion was introduced in 1934 by the French mathematicians Marty, at the 8th Congress of Scandinavian Mathematicians. The motivating example was the quotient of a group by any, not necessary normal, subgroup. Algebraic hyperstructures in the sense of Marty are a suitable generalization of classical algebraicstructures. In a classical algebraic structure, the composition of two elements isan element, while in an algebraic hyperstructure, the composition of two elements isa set. Many papers and several books have been written till now on hyperstructures [2, 3, 4, 14, 32]. Many of them are dedicated to the applications of hyperstructures in other disciplines. In 1996, Santilli and Vougiouklis [24] point out that inphysics the most interesting hyperstructures are the one called e-hyperstructures. The e-hyperstructures are special kind of hyperstructures and they can be interpreted asa generalization of two important concepts for physics: Isotopies and

Genotopies. In [15], Davvaz, Santilli and Vougiouklis studied multi-valued hyperstructures following the apparent existence in nature of a realization of two-valued hyperstructures with hyperunits characterized by matter-antimatter systems and their extensions, where matter is represented with conventional mathematics and antimatter is represented with isodual mathematics, also see [16]. In [17], the authors presented Ying's twin universes, Santilli's isodual theory of antimatter, and Davvaz–Santilli–Vougiouklis two-valued hyperstructures representing matter and antimatter in two distinct but co-existing space times. They identified a seemingly new map for both matter and antimatter providing a mathematical prediction of Ying's twin universes, and introduced a four-fold hyperstructure representing matter-antimatter as well as Ying's twin universes, all co-existing in distinct space times. Another motivation for the study of hyperstructures comes from physical phenomenon as the nuclear fission. This motivation and the results were presented by Hošková, Chvalina and Račková (see [20, 21]). In [11], the authors provided, for the first time, a physical example of hyperstructures associated with the elementary particle physics, Leptons. They have considered this important group of the elementary particles and shown that this set along with the interactions between its members can be described by the algebraic hyperstructures.

Mendel, the father of genetics took the first steps in defining "contrasting characters, genotypes in $F1$ and $F2$ . . . and setting different laws". The genotypes of $F2$ is dependent on the type of its parents genotype and it follows certain roles. In [18], Ghadiri et al. analyzed the second generation genotypes of monohybrid and a dihybrid with a mathematical structure. They used the concept of H_v-semigroup structure in the $F2$-genotypes with cross operation and proved that this is an H_v-semigroup. They determined the kinds of number of the H_v-subsemigroups of $F2$-genotypes. In [10], inheritance issue based on genetic information is looked at carefully via a new hyperalgebraic approach. Several examples are provided from different biology points of view, and it is shown that the theory of hyperstructures exactly fits the inheritance issue.

Another motivation for the study of hyperstructures comes from chemical reactions. In [6], Davvaz and Dehghan-Nezhad provided examples of hyperstructures associated with chain reactions. In [7], Davvaz et al. introduced examples of weak hyperstructures associated with dismutation reactions. In [12], Davvaz et al. investigated the examples of hyperstructures and weak hyperstructures associated with redox reactions, see [1, 8, 9, 13].

2. WEAK ALGEBRAIC HYPERSTRUCTURES

Weak hyperstructures or H_v-structures were introduced by Vougiouklis at the Fourth AHA congress (1990) [28]. The concept of an H_v-structure constitutes a generalization of the well-known algebraic hyperstructures (smihypergroup, hypergroup, hyperring and so on). Actually some axioms concerning the above hyperstructures such as the associative law, the distributive law and so on are replaced by their corresponding weak axioms. Since then the study of H_v-structure theory has been pursued in many directions by Vougiouklis, Davvaz, Spartalis, Dramalidis, Hošková, and others. In this section, we present some definitions and basic facts about weak hyperstructures [5, 29, 31].

Let H be a non-empty set and $\cdot : H \times H \to \rho^*(H)$ be a hyperoperation. The "$\cdot$" in H is called *weak associative* if

$$x\cdot (y \cdot z) \cap (x \cdot y) \cdot z \neq \emptyset, \text{ for all } x, y, z \in H.$$

The " $\cdot$" is called *weak commutative* if

$$x \cdot y \cap y \cdot x \neq \phi, for\, all\, x, y \in H.$$

The " $\cdot$" is called *strongly commutative* if

$$x \cdot y = y \cdot x, for\ all\ x, y \in H.$$

The hyperstructure $(H, \cdot)$ is called an *H_v-semigroup* if "$\cdot$ " is weak associative. An H_v-semigroup is called an *H_v-group* if

$$a \cdot H = H \cdot a = H, for\, all\, a \in H.$$

In an obvious way, the H_v-subgroup of an H_v-group is defined.

Consider $H = \{e, a, b, c\}$ and define $*$ on H with the help of the following table:

*	*e*	*a*	*b*	*C*
e	*e*	*a*	*b*	*c*
a	*a*	*e,a*	*c*	*b*
b	*b*	*c*	*e,b*	*a*
c	*c*	*b*	*a*	*e,c*

Then $(H, *)$ is an H_v-group which is not a hypergroup. Indeed, we have

$$(a * b) * c = c * c = \{e,c\},\ a * (b * c) = a * a = \{e, a\}.$$

Therefore, $*$is not associative.

A first motivation to study the weak hyperstructures is the following example.
Let $(G, \cdot)$ be a group and R be an equivalence relation on G. In G/R consider the hyperoperation $\odot$ defined by $x \odot y = \{z \mid z \in x \cdot y\}$, where x denotes theequivalence class of the element x. Then, $(G, \odot)$ is an H_v-group which is not always a hypergroup.

All the weak properties for hyperstructures can be applied for subsets. Forexample, if $(H, \cdot)$ is a weak commutative H_v-group, then for all non-empty subsets A, B, C of H, we have $(A \cdot B) \cap (B \cdot A) \neq \phi\, and\ A \cdot (B \cdot C) \cap (A \cdot B) \cdot C \neq \phi$. To prove this, one has simply to take one element of each set.

Let $(H_1, \cdot), (H_2, *)$ be two H_v-groups. A map $f : H_1 \to H_2$ is called an *H_v-homomorphism* or a *weak homomorphism* if

$$f(x \cdot y) \cap f(x) * f(y) \neq \phi, for\, all\, x, y \in H_1.$$

f is called an *inclusion homomorphism* if

$$f(x \cdot y) \subseteq f(x) * f(y), \textit{ for all } x, y \in H_1.$$

Finally, f is called a *strong homomorphism* or a *good homomorphism* if

$$f(x \cdot y) = f(x) * f(y), \textit{ for all } x, y \in H_1.$$

If f is onto, one to one and strong homomorphism, then it is called an *isomorphism.* Moreover, if the domain and the range of f are the same H_v-group, then the isomorphism is called an *automorphism*. We can easily verify that the set of all automorphisms of H, defined by *Aut H*, is a group.

Several H_v-structures can be defined on a set H. A partial order on these hyperstructures can be introduced, as follows.

Let $(H, \cdot)$ and $(H, *)$ be two H_v-groups defined on the same set H. We say that " $\cdot$" less than or equal to "$*$" and we write $\cdot \leq *$, if there is $f \in Aut(H, *)$ such that $x \cdot y \subseteq f(x*y)$, for all $x, y \in H$. If a hyperoperation is weak associative, then every greater hyperoperation, defined on the same set is also weak associative. In [30], the set of all *Hv*-groups with a scalar unit defined on a set with three elements is determined using this property.

Greater hyperoperation from the one of a given *Hv*-group defines an H_v-group. The weak commutativity is also valid for every greater hyperoperation. We remarkthat this statement is not true for hypergroups.

Let $(H, \cdot)$ be an H_v-group. The relation β^* is the smallest equivalence relation on H such that the quotient H/β^* is a group. β^* is called the *fundamental equivalence relation* on H. If U denotes the set of all finite products of elements of H, then a relation β can be defined on H whose transitive closure is the fundamental relationβ^*. The relation β is defined as follows: for x and y in H we write $x\beta y$ if and only if $\{x, y\} \subseteq u$ for some $u \in U$. We can rewrite the definition of β^*on H as follows: $a\beta^* b$if and only if there exist $z_1, \ldots, z_{n+1} \in H$ with $z_1 = a$, $z_{n+1} = b$ and $u_1, \ldots, u_n \in U$ such that $\{z_i, z_{i+1}\} \subseteq u_i$ $(i = 1, \ldots, n)$. Suppose that $\beta^*(a)$ is the equivalence classcontaining $a \in H$. Then, the product $\odot$ on H/β^* is defined as follows:

$$\beta^*(a) \odot \beta^*(b) = \{\beta^*(c) | \ c \in \beta^*(a) \cdot \beta^*(b)\} \text{ for all } a, b \in H.$$

It is not difficult to see that $\beta^*(a) \odot \beta^*(b)$ is the singleton $\{\beta^*(c)\}$ for all $c \in \beta^*(a) \cdot \beta^*(b)$. In this way H/β^* becomes a group.

Let $(H, \cdot)$ be an H_v-group. An element $x \in H$ is called *single* if its fundamental class is singleton, i.e., $\beta^*(x) = \{x\}$. Denote by SH the set of all single elements of H. Let $(H, \cdot)$ be an H_v-group and $x \in SH$. Let $a \in H$ and take any element $v \in H$ such that $x \in a \cdot v$. Then,

$$\beta^*(a) = \{h \in H \mid h \cdot v = x\}.$$

Suppose that $(H,\cdot)$ is an H_v-group such that SH is non-empty. Then, the only greater hyperoperations $\cdot < *$ for which the H_v-groups $(H,*)$ contain single elements are the ones with the same fundamental group, since the fundamental classes aredetermined from the products of a single element with the elements of the group. On the other hand, a less hyperoperation $\circ < \cdot$ can have the same set SH if only in the products of non-single elements the $\circ$ is less than $\cdot$. Finally, if ρ and σ areequivalence relations with $\rho < \sigma$ such that H/ρ and H/σ are non-equal groups, then they can not have both single elements.

Let $(H, \cdot)$ be an H_v-group with (left, right) identity elements. Then, H is called (*left, right*) *reversible* in itself when any relation $c \in a \cdot b$ implies the existence of a left inverse a' of a and a right inverse b' of b such that $b \in a' \cdot c$ and $a \in c \cdot b'$. The H_v-group $(H, \cdot)$ is called *feebly quasi-canonical* if it is regular, reversible and satisfies the following conditions:

For each $a \in H$, if a', a'' are inverses of a, then for each $x \in H$, we have:

$$a' \cdot x = a'' \cdot x \text{ and } x \cdot a' = x \cdot a''.$$

A feebly quasi-canonical H_v-group H is called *feebly canonical* if it is strongly commutative.

3. CHEMICAL REACTIONS

Chemistry is the study of matter and of the changes matter undergoes. A chemicalequation describes the products of a reaction that from the starting molecules oratoms. Chemistry seeks to predict the products that result from the reaction of specificquantities of atoms or molecules. Chemists accomplish this task by writing andbalancing chemical equations. Symmetry is very important in chemistry researchesand group theory is the tool that is used to determine symmetry. Classical algebraicstructures (group theory) is a mathematical method by which aspects of a moleculessymmetry can be determined. Algebraic hyperstructures are generalizations of classical algebraic structures. In a classical algebraic structure, the composition of twoelements is a set. A motivation for the study of hyperstructures comes from chemical reactions. In [6], Davvaz and Dehghan-Nezhad provided examples of hyperstructures associated with chain reactions. In [7], Davvaz et al. introduced examples of weak hyperstructures associated with dismutation reactions. In [12], Davvaz et al. investigated the examples of hyperstructures and weak hyperstructures associated withredox reactions. In this section we review these examples. For more details we referto [6, 7, 12].

3.1 CHAIN REACTIONS

Chain reaction, in chemistry and physics, process yielding products that initiatefurther processes of the same kind, a self-sustaining sequence. Examples from chemistry are burning a fuel gas, the development of rancidity in fats, “knock” in internalcombustionengines, and the polymerization of ethylene to polyethylene. The bestknownexamples in physics are nuclear fissions brought about by neutrons. Chainreactions are in general very rapid but are also highly sensitive to reaction conditions,probably because the substances that sustain the reaction are easily affectedby substances other than the reactants themselves. An atom or group of atomspossessing an odd (unpaired) electron is called radical. Radical species can be electricallyneutral, in which case they are sometimes referred to as free radicals. Pairsof electrically neutral "free" radicals are formed via homolytic bond breakage. Thiscan be achieved by heating in non-polar solvents or the vapor phase. At elevatedtemperature or under the influence ultraviolet light at room temperature, all molecularspecies will dissociate into radicals. Homolsis or homolytic bond fragmentationoccurs when (in the language of Lewis theory) a two electron covalent bond breaks and one electron goes to each of the partner species.

For example, chlorine, Cl_2, forms chlorine radicals ($Cl^{\bullet}$) and peroxides form oxygenradicals.

$$X—X \to 2X^{\bullet}$$
$$Cl—Cl \to 2\ Cl^{\bullet}$$
$$R—O—O–R \to R—O^{\bullet}$$

Radical bond forming reactions (radical couplings) are rather rare processes. The reason is because radicals are normally present at low concentrations in a reactionmedium, and it is statistically more likely they will abstract a hydrogen, or undergoanother type of a substitution process, rather than reacting with each other by coupling.And as radicals are uncharged, there is little long range columbic attraction between two radical centers. Radical substitution reactions tend to proceed as chainreaction processes, often with many thousands of identical propagation steps. Thepropensity for chain reactivity gives radical chemistry a distinct feel compared withpolar Lewis acid/base chemistry where chain reactions are less common. Methanecan be chlorinated with chlorine to give chloromethane and hydrogen chloride. Thereaction proceeds as a chain, radical, substitution mechanism. The process is a littlemore involved, and three steps are involved: initiation, propagation and termination:

(1) $Cl_2 \to 2Cl^{\bullet}$

(1) is called chain-initiating step.

(2) $Cl^{\bullet} + CH_4 \to HCl + CH^{\bullet}_3$

(3) $CH^{\bullet}_3 + Cl_2 \to CH_3Cl + Cl^{\bullet}$

then (2), (3), (2), (3), etc, until finally:

(2) and (3) are called chain-propagating steps.

(4) $Cl^{\bullet} + Cl^{\bullet} \to Cl_2$ or

(5) $CH^{\bullet}_3 + CH^{\bullet}_3 \to CH_3CH_3$ or

(6) $CH^{\bullet}_3 + Cl^{\bullet} \to CH_3Cl.$

(4), (5) and (6) are called chain-terminating steps.

First in the chain of reactions is a chain-initiating step, in which energy is absorbedand a reactive particle generated; in the present reaction it is the cleavage of chlorineinto atoms (Step 1).There are one or more chain-propagating steps, each of which consumes a reactive particle and generates another; there they are the reaction ofchlorine atoms with methane (Step 2), and of methyl radicals with chlorine (Step3).

A chlorine radical abstracts a hydrogen from methane to give hydrogen chloride and a methyl radical. The methyl radical then abstracts a chlorine atom (a chlorineradical) from $Cl2$ to give methyl chloride and a chlorine radical... which abstractsa hydrogen from methane... and the cycle continues... Finally there are chainterminatingsteps, in which reactive particles are consumed but not generated; inthe chlorination of methane these would involve the union of two of the reactiveparticles, or the capture of one of them by the walls of the reaction vessel.

The halogens are all typical non-metals. Although their physical forms differfluorine and chlorine are gases, bromine is a liquid and iodine is a solid at room temperature,each consists of diatomic molecules; F_2,Cl_2,Br_2 and I_2. The halogens all react with hydrogen to form gaseous compounds, with the formulas HF, HCl, HBr and HI all of which are very soluble in water. The halogens all react with metals to give halides.

$$:\overset{\cdot\cdot}{\underset{\cdot\cdot}{F}} - \overset{\cdot\cdot}{\underset{\cdot\cdot}{F}}:, \quad :\overset{\cdot\cdot}{\underset{\cdot\cdot}{Cl}} - \overset{\cdot\cdot}{\underset{\cdot\cdot}{Cl}}:, \quad :\overset{\cdot\cdot}{\underset{\cdot\cdot}{Br}} - \overset{\cdot\cdot}{\underset{\cdot\cdot}{Br}}:, \quad :\overset{\cdot\cdot}{\underset{\cdot\cdot}{I}} - \overset{\cdot\cdot}{\underset{\cdot\cdot}{I}}:$$

The reader will find in [22] a deep discussion of chain reactions and halogens. During chain reaction

$$A_2 + B_2 \overset{heat\ or\ light}{\longleftrightarrow} 2AB$$

there exist all molecules A_2, B_2, AB and whose fragment parts $A^{\bullet}$, $B^{\bullet}$ in experiment. The elements of this collection can by combine with each other. All combinational probabilityfor the set $H = \{A^{\bullet}, B^{\bullet}, A_2, B_2, AB\}$ to do without energy can be displayed as in Table 1.

Table 1. Chain Reactions.

$\oplus$	A^*	B^*	A_2	B_2	AB
A^*	A^*, A_2	A^*,B^*, AB	A^*, A_2	A^*,B_2, B^*,AB	A^*,AB,A_2,B^*
B^*	A^*, B^*,AB	B^*,B_2	A^*,B^*,AB, A_2	B^*,B_2	A^*,B^*,AB,B_2
A_2	A^*, A_2	A^*,B^*,AB, A_2	A^*, A_2	A^*,B^*,A_2,B_2,AB	A^*,B^*,A_2,AB
B_2	A^*,B^*,B_2, AB	B^*,B_2	A^*,B^*,A_2,B_2,AB	B^*,B_2	A^*,B^*,B_2,AB
AB	A^*,AB,A_2,B^*	A^*,B^*,AB,B_2	A^*,B^*,A_2,AB	A^*,B^*,B_2,AB	A^*,B^*,A_2,B_2,AB

Then, $(H,\oplus)$ is an H_v-group [6]. Moreover, $X = \{A^\bullet, A_2\}$ and $Y = \{B^\bullet, B_2\}$ are only H_v-subgroups of $(H,\oplus)$ [6]. If we consider $A = H$ and $B \in \{F, CL, Br, I\}$ (for example $B = I$), the complete reactions table becomes Table 2.

Table 2. For *H* and *I*.

$\oplus$	$H^\bullet$	$I^\bullet$	H_2	I_2	HI
H°	$H^\bullet, H_2$	$H^\bullet, I^\bullet, HI$	$H^\bullet, H_2$	$H^\bullet, I_2, I^\bullet, HI$	$H^\bullet, HI, H_2, I^\bullet$
$I^\bullet$	$H^\bullet, I^\bullet, HI$	$I^\bullet, I_2$	$H^\bullet, I^\bullet, HI, H_2$	$I^\bullet, I_2$	$H^\bullet, I^\bullet, HI, I_2$
H_2	$H^\bullet, H_2$	$H^\bullet, I^\bullet, HI, I_2$	$H^\bullet, H_2$	$H^\bullet, I^\bullet, H_2, I_2, HI$	$H^\bullet, I^\bullet, H_2, HI$
I_2	$H^\bullet, I^\bullet, I_2, HI$	$H^\bullet, I_2$	$H^\bullet, I^\bullet, H_2, I_2, HI$	$H^\bullet, I_2$	$H^\bullet, I^\bullet, I_2, HI$
HI	$H^\bullet, HI, H_2, I^\bullet$	$H^\bullet, I^\bullet, HI, I_2$	$H^\bullet, I^\bullet, H_2, HI$	$H^\bullet, I^\bullet, H_2, HI$	$H^\bullet, I^\bullet, H_2, I_2, HI$

3.2 DISMUTATION REACTIONS

In a redox reactions or oxidation-reduction reaction, electrons are transferred fromone reactant to another. Oxidation refers to the loss of electrons, while reductionrefers to the gain of electrons. A substance that has strong affinity for electronsand tends to extract them from other species is called an oxidizing agent or an oxidant. A reducing agent, or reductant, is a reagent that readily donates electronsto another species [26]. A half reaction is a reduction or an oxidation reaction. Twohalf-reactions are needed to form a whole reaction. Redox reactions have a numberof similarities to acid-base reactions. Like acid-base reactions, redox reactions area matched set; you don't have an oxidation reaction without a reduction reactionhappening at the same time. When the change in free energy (ΔG) is negative, aprocess or chemical reaction proceeds spontaneously in the forward direction. WhenΔG is positive, the process proceeds spontaneously in reverse. In electrochemical reactions $\Delta G = -nFE$, where n, F and E are number of electrons transferred inthe reaction, Faraday constant and cell potential, respectively [26].

The change in the oxidation state of a species lets you know if it has undergone oxidation or reduction. Oxidation is the process in which an atom undergoes an algebraic increase in oxidation number, and reduction is the process in which anatom undergoes an algebraic decrease in oxidation number. On this basis, oxidationreduction is involved in the reaction;

$$O_2 + C \rightarrow CO_2$$

In the reaction, oxidation number of the C atom increases from zero to +4 whereas, the oxidation number of O atom decreases from zero to −2. Furthermore, the total increase in the oxidation number equals to the total decrease in oxidation number [23].

Disproportion at ion or dismutation is used to describe two particular types of chemical reaction:

(1) A chemical reaction of the type $2A \rightarrow A' + A''$, where A, A' and A'' are different chemical pieces [27]. Most but not all are redox reactions. For example $2H_2 \rightarrow OH_3O^+ + OH^-$ is a *disproportionation*, but is not a redox reaction.

(2) A chemical reaction in which two or more atoms of the same element originallyhaving the same oxidation state react with other chemical(s) or themselves to give different oxidation numbers. In another word, disproportionation is a reaction in which a species is simultaneously reduced and oxidized to form two differentoxidation numbers. The reverse of disproportionation is called comproportionation. *Comproportionation* is a chemical reaction where two reactants, each containing thesame element but with a different oxidation number, will form a product with anoxidation number intermediate of the two reactants. For example, an element tin in the oxidation states 0 and +4 can comproportionate to the state +2. The standardreduction potentials of all half reactions are:

$$E^\circ_{Sn^{4+}/Sn^{2+}} = 0.154\ V, E^\circ_{Sn^{2+}/Sn} = -0.136\ V, E^\circ_{Sn^{4+}/Sn} = 0.009V$$

Therefore, the comproportionation reaction is spontaneous.

$$Sn + Sn^{4+} \rightarrow 2Sn^{2+}$$

All combinational probability for the set $S = \{Sn, Sn^{2+}, Sn^{4+}\}$ to do withoutenergy can be displayed as follows. The major products are written in Table 3.

Table 3.Dismutation Reactions S_n.

$\oplus$	Sn	Sn^{2+}	Sn^{4+}
Sn	Sn	Sn, Sn^{2+}	Sn^{2+}
Sn^{2+}	Sn, Sn^{2+}	Sn^{2+}	Sn^{2+}, Sn^{4+}
Sn^{4+}	Sn^{2+}	Sn^{2+}, Sn^{4+}	Sn^{4+}

Then, $(S, \oplus)$ is weak associative. Also, we can conclude that $(\{S_n, S_n^{2+}\}, \oplus)$ is a hypergroup and $(\{Sn^{2+}, Sn^{4+}\}, \oplus)$ is an H_v-semigroup [7]. Chlorine gas reacts with dilute hydroxide to form chloride, chlorate and water.The ionic equation for this reaction is as follows [19]:

$$3Cl_2 + 6OH^- \rightarrow 5Cl^- + ClO_3^- + 3H_2O$$

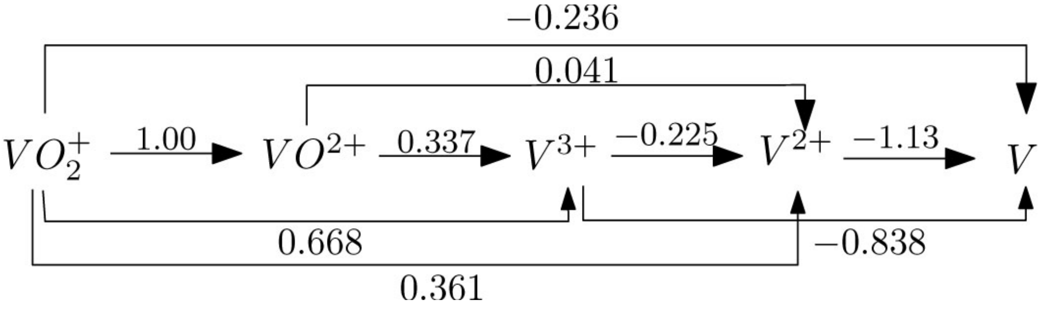
As a reactant, the oxidation number of the elemental chlorine, chloride and chlorate are 0, 1 and +5, respectively. Therefore, chlorine has been oxidized to chlorate whereas; it has been reduced to chloride [19].

Indium has three oxidation states 0,+1 and +3. The standard reduction potentialsof all half reactions are: $E^{\circ}_{In^{3+}/In^{+}} = -0.434V$, $E^{\circ}_{In^{+}/In} = -0.147V$, $E^{\circ}_{In^{3+}/In} = -0.338V$. According to the standard reduction potentials, disproportionationreaction of In^{+} is spontaneous. All combinational probability for the set $S = \{In, In^{+}, In^{3+}\}$ to do without energy can be displayed as Table 4.

Table 4. Dismutation Reactions *In*.

$\oplus$	In	In^{+}	In^{3+}
In	In	In, In^{+}	In, In^{3+}
In^{+}	In, In^{+}	In, In^{3+}	In^{+}, In^{3+}
In^{3+}	In, In^{3+}	In^{+}, In^{3+}	In^{3+}

Then, $(S,\oplus)$ is weak associative. Clearly, $\oplus$is commutative. Also, the reproduction axiom holds. Therefore, $(S,\oplus)$ is a commutative H_v-group [7]. Vanadium forms a number of different ions including V, V^{2+}, V^{3+}, VO^{2+} and VO^{2+}. The oxidation states of these species are 0, +2, +3, +4 and +5, respectively. The standard reduction potentials of all corresponding half reactions are:



All combinational probability for the set $S = \{V, V^{2+}, V^{3+}, VO^{2+}, VO^{2+}\}$to dowithout energy in acidic media can be displayed as following table. When thereactants are added in appropriate stoichiometric ratios. For example vanadium(V) reacts with VO^{2+} as follows:

$$2V+3VO_2^{+}+12H^{+}\to 5V^{3+}+6H_2O$$

Then, $(S,\oplus)$ is a hyperstructure. The hyperstructures

$$(\{V, V^{2+}\},\oplus),\ (\{V^{2+}, V^{3+}\},\oplus),(\{V^{3+}, VO^{2+}\},\oplus) \text{ and } (\{VO^{2+}, VO^{+2}\},\oplus)$$

are hypergroups [7]. Moreover, we have:

$$(\{V, V^{2+}\},\oplus)\cong(\{V^{2+}, V^{3+}\},\oplus)\cong(\{V^{3+}, VO^{2+}\},\oplus)\cong(\{VO^{2+}, VO^{2+}\},\oplus).$$

The major products between all forms of vanadium are showed in Table 6. It is assumed the reactants are added together in 1 : 1 mole ratios.

Table 5. Vanadium.

$\oplus$	V	V^{2+}	V^{3+}	VO^{2+}	VO_2^{+}
V	V	V, V^{2+}	V^{2+}	V^{2+}, V^{3+}	V^{3+}
V^{2+}	V, V^{2+}	V^{2+}	V^{2+}, V^{3+}	V^{3+}	V^{3+}, VO^{2+}
V^{3+}	V^{2+}	V^{2+}, V^{3+}	V^{3+}	V^{3+}, V^{2+}	VO^{2+}
VO^{2+}	V^{2+}, V^{3+}	V^{3+}	V^{3+}, VO^{2+}	VO^{2+}	VO^{2+}, VO_2^{+}
VO_2^{+}	V^{3+}	V^{3+}, VO^{2+}	VO^{2+}	VO^{2+}, VO_2^{+}	VO_2^{+}

Table 6.The Major Products between all Forms of Vanadium.

$\oplus$	V	V^{2+}	V^{3+}	VO^{2+}	VO_2^{+}
V	V	V, V^{2+}	V, V^{2+}	V^{2+}	V^{2+}, V^{3+}
V^{2+}	V, V^{2+}	V^{2+}	V^{2+}, V^{3+}	V^{3+}	V^{3+}, VO^{2+}
V^{3+}	V, V^{2+}	V^{2+}, V^{3+}	V^{3+}	V^{3+}, VO^{2+}	VO^{2+}
VO^{2+}	V^{2+}	V^{3+}	V^{3+}, VO^{2+}	VO^{2+}	VO^{2+}, VO_2^{+}
VO_2^{+}	V^{2+}, V^{3+}	V^{3+}, VO^{2+}	VO^{2+}	VO^{2+}, VO_2^{+}	VO_2^{+}

Therefore, $(S,\oplus)$ is a hyperstructure. The hyperstructures

$$(\{V, V^{2+}\},\oplus), (\{V^{2+}, V^{3+}\},\oplus),(\{V^{3+}, VO^{2+}\},\oplus) \text{ and } (\{VO^{2+}, VO^{+2}\},\oplus)$$

are hypergroups. Moreover, we have:

$$(\{V, V^{2+}\},\oplus) \cong (\{V^{2+}, V^{3+}\},\oplus)\cong(\{V^{3+}, VO^{2+}\},\oplus)\cong(\{VO^{2+}, VO^{2+}\},\oplus).$$

3.3 REDOX REACTIONS

Redox (reduction-oxidation) reactions include all chemical reactions in which atomshave their oxidation state changed. This can be either a simple redox process, such as the oxidation of carbon to yield carbon dioxide (CO_2) or the reduction of carbon by hydrogen to yield methane (CH_4), or a complex process such as the oxidation of glucose ($C_6H_{12}O_6$) in the human body through a series of complexelectron transfer processes. Oxidation is the loss of electrons or an increase in oxidation state, and reduction is the gain of electrons or a decrease in oxidation state by an analyte (molecule, atom or ion). There can not be an oxidation reactionwithout a reduction reaction happening simultaneously. Therefore the oxidation alone and the reduction alone are each called a half-reaction, because two half reactions always occur together to form a whole reaction [23].

Each half-reaction has a standard reduction potential (E^0), which is equal to thepotential difference at equilibrium under standard conditions of an electrochemicalcell in which the cathode reaction is the half-reaction considered, and the anode isa standard hydrogen electrode (SHE). For a redox reaction, the potential of the cellis defined by: $E^0_{cell} = E^0_{cathode} - E^0_{anode}$. If the potential of a redox reaction (E^0_{cell})is positive, this reaction will spontaneous [23]. For example, consider the redox reaction of Ag^{2+} with Ag:

$$Ag^{2+} + Ag \rightarrow Ag^{+}.$$

We can write two half-reactions for this reaction:

(1) $Ag^{2+} + e \rightarrow Ag^{+}$,

(2) $Ag \rightarrow Ag^{+} + e$.

The E^0 of the first reaction ($E^0_{cathode}$) is 1.98 V (vs. SHE) and the $E^0$of the second reaction (E^0_{anode}) is 0.799 V (vs. SHE) [26]. Therefore, in this case, the $E^0_{cell}(E^0_{cathode} - E^0_{anode} = 1.181)$ is positive and the above redox reaction between $Ag^{2+}$and Ag is spontaneous. Silver (Ag) is a transition metal and has a large number of applications in jewelry, electrical contacts and conductors, catalysis of chemical reactions, disinfectants and microbiocides. Silver plays no known natural biological role in humans and itself is not toxic, but most silver salts are toxic, and some may be carcinogenic. Ag can be in three oxidation state: Ag (0), Ag (I) and Ag (II). Among Ag (I) andAg (II), Ag (I) is very well characterized and many simple ionic compounds areknown containing Ag^{+}. However, AgF_2 is known which Ag has oxidation state ofII in it. AgF_2 is strongly oxidizing and a good fluorimating agent. But Ag (II) ismore stable in complex forms. A number of Ag (II) complexes have been obtainedby oxidation of Ag (I) salts is aqueous solution in the presences of the ligand. Forexample, $[Ag\ (pyridine)_4]^{2+}$ and $[Ag\ (bi\ pyridine)_2]^{2+}$ are quite stable. The +1oxidation state is the best known oxidation state of silver. Ag^{+} salts are generallyinsoluble in water with the exception of nitrate, fluoride and perchlorate. Moststable Ag (I) complexes have a linear structure [25].

As described above, Ag species with different oxidation state can react with themselves. All possible products for spontaneous reactions are presented in Table7.

Table 7.Redox Reactions Ag.

$\oplus$	Ag^{2+}	Ag^{+}	Ag
Ag^{2+}	Ag^{2+}	Ag^{+}, Ag^{2+}	Ag^{+}
Ag^{+}	Ag^{+}, Ag^{2+}	Ag^{+}	Ag, Ag^{+}
Ag	Ag^{+}	Ag^{+}, Ag	Ag

The Table 7 is isomorphic to Table 3 of dismutation reactions. Therefore, $\oplus$ is weak associative. Also, we conclude that $(\{Ag^{2+}, Ag^{+}\}, \oplus)$ and $(\{Ag^{+}, Ag\}, \oplus)$ are hypergroups.

Copper (Cu) is a ductile metal with very high thermal and electrical conductivity. It is used as a conductor of heat and electricity, a building material, anda constituent of various metal alloys. Cu can be in four oxidation state: Cu (0),$Cu(I)$, Cu (II) and Cu (III). In nature, copper mainly is as $CuFeS_2$, with oxidationstate of II for Cu. Also, Cu can be as Cu_2S or Cu_2O with the oxidation stateof I. Pure copper is obtained by electrolytic refining using sheets of pure copper as cathode and impure copper as anode. In this process different ions of Cu, $Cu(II)$or $Cu(I)$, reduced to $Cu(0)$ at cathode. $Cu(III)$ is generally uncommon, howeversome its complexes are known [25].

The standard reduction potential (E^0) for conversion of each oxidation state to another are: E^0 (Cu^{3+}/Cu^{2+}) = 2.4 V ,E^0 (Cu^{2+}/Cu^{+})= 0.153 V , E^0 (Cu^{2+}/Cu)=0.342 V and E^0 (Cu^{+}/Cu)= 0.521 V , where potentials are versus SHE [26]. Accordingto these standard potentials, and similar to example of Ag, the following reactions are spontaneous:

(1) $Cu^{3+} + Cu^{+} \to Cu^{2+}$,

(2) $Cu^{3+} + Cu \to Cu^{2+} + Cu^{+}$.

Therefore, all possible products in reactions between oxidation states of Cu which can be produced spontaneously are listed in Table 8.

Table 8.Redox Reactions *Cu*.

$\odot$	Cu	Cu^{+}	Cu^{2+}	Cu^{3+}
Cu	Cu	Cu, Cu^{+}	Cu^{2+}, Cu	Cu^{2+}, Cu^{+}
Cu^{+}	Cu, Cu^{+}	Cu^{+}	Cu^{2+}, Cu^{+}	Cu^{2+}
Cu^{2+}	Cu, Cu^{2+}	Cu^{2+}, Cu^{+}	Cu^{2+}	Cu^{2+}, Cu^{3+}
Cu^{3+}	Cu^{+}, Cu^{2+}	Cu^{2+}	Cu^{2+}, Cu^{3+}	Cu^{3+}

In Table 8, the hyperoperation $\odot$ is weak associative. Hence, we have an H_v-semigroup. The hyperstructures

$(\{Cu, Cu^{+}\},\odot)$, $(\{Cu, Cu^{2+}\},\odot)$, $(\{Cu^{+}, Cu^{2+}\},\odot)$ and $(\{Cu^{2+}, Cu^{3+}\},\odot)$

are hypergroups. Let H be a set with three elements. OnH, we define the following hyperoperation: $x \star y = \{x, y\}$, for all $x, y \in H$.

It is easy to see that $\star$is associative and so $(H, \star)$ is a hypergroup. Now, we have

$$(\{Cu, Cu^{+}, Cu^{2+}\},\odot)\cong(H, \star).$$

Note that $(\{Cu^{+}, Cu^{2+}, Cu^{3+}\},\odot)$ is not semihypergroup. Americium (Am) is a transuranic radioactive chemical element in actinide series. It has four oxidation states of 0, 2, 3 and 4. The standard reduction potential (E0) for conversion of each oxidation state to another are: E0 (Am^{4+}/Am^{3+})= 2.6 V, E^0 (Am^{3+}/Am^{2+}) = −2.3 V, E^0 (Am^{3+}/Am) = −2.048 V and E^0 (Am^{2+}/Am)=−1.9 V , where potentials are versus SHE [26]. Therefore, the following reaction is spontaneous:

$$Am^{4+} + Am^{2+} \to Am^{3+}.$$

Therefore, all possible combinations for different oxidation states of *Am* which can be produced without energy are presented in Table 9.

Table 9. Redox Reactions *Am*.

$\otimes$	Am	Am^{2+}	Am^{3+}	Am^{4+}
Am	Am	Am, Am^{2+}	Am, Am^{3+}	Am, Am^{4+}
Am^{2+}	Am, Am^{2+}	Am^{2+}	Am^{2+}, Am^{3+}	Am^{3+}
Am^{3+}	Am, Am^{3+}	Am^{2+}, Am^{3+}	Am^{3+}	Am^{3+}, Am^{4+}
Am^{4+}	Am, Am^{4+}	Am^{3+}	Am^{3+}, Am^{4+}	Am^{4+}

Regarding to Table 9, similar to Table 8, we have

$$(\{Am, Am^{2+}, Am^{3+}\},\otimes)\cong(H, \star).$$

Note that $(\{Am^{2+}, Am^{3+}, Am^{4+}\},\otimes)$ is not semihypergroup.

Gold (*Au*) is a dense, soft, shiny, malleable and ductile metal and can be in fouroxidation states of *Au* (0), *Au* (*I*), *Au* (*II*) and *Au* (*III*). *Au* (*III*) is common forgold compounds and exist as: Au_2O_3, AuF_3, $AuCl_3$, $AuBr_3$ and $Au(OH)_3$. *Au* (*I*)is much less stable in solution and is stabilized in complexes [25].

The standard reduction potential (E^0) for conversion of each oxidation state toanother are: $E^0\,(Au^{3+}/Au^{+})= 1.401\ V$, $E^0\,(Au^{3+}/Au)= 1.498\ V$, $E^0\,(Au^{2+}/Au^{+})=1.8\ V$ and $E^0\,(Au^{+}/Au)= 1.692\ V$, where potentials are versus SHE [26]. Accordingto these standard potentials, the following reaction is spontaneous:

$$Au^{2+} + Au \rightarrow Au^{+}.$$

Therefore, the major products in reactions between oxidation states of Au whichcan be produced spontaneously are listed in Table 10.

Table 10. Redox Reactions *Au*.

$\uplus$	Au	Au^{+}	Au^{2+}	Au^{3+}
Au	Au	Au, Au^{+}	Au^{+}	Au, Au^{3+}
Au^{+}	Au, Au^{+}	Au^{+}	Au^{+}, Au^{2+}	Au^{+}, Au^{3+}
Au^{2+}	Au^{+}	Au^{+}, Au^{2+}	Au^{2+}	Au^{2+}, Au^{3+}
Au^{3+}	Au, Au^{3+}	Au^{+}, Au^{3+}	Au^{2+}, Au^{3+}	Au^{3+}

The H_v-semigroups defined in Tables 9 and 10 are isomorphic.

REFERENCES

1. S. C. Chung, K. M. Chun, N. J. Kim, S. Y. Jeong, H. Sim, J. Lee and H. Maeng, Chemical hyperalgebras for three consecutive oxidation states of elements, *MATCH Communications in Mathematical and in Computer Chemistry* **72** (2014) 389–402.
2. P. Corsini, Prolegomena of Hypergroup Theory, Second edition, Aviani Editore, 1993.
3. P. Corsini and V. Leoreanu, Applications of Hyperstructures Theory, *Advances in Mathematics, Kluwer Academic Publisher*, 2003.
4. B. Davvaz, Polygroup Theory and Related Systems, World Scientific Publishing Co. Pte. Ltd., Hackensack, NJ, 2013.
5. B. Davvaz, A brief survey of the theory of H_V-structures, Proc. 8*th* International Congress on Algebraic Hyperstructures and Applications, 1−9 September, 2002, Samothraki, Greece, *Spanidis Press*, 2003, 39–70.
6. B. Davvaz and A. Dehgan-Nezad, Chemical examples in hypergroups, *Ratio Matematica* **14** (2003) 71–74.
7. B. Davvaz, A. Dehghan-Nezhad and A. Benvidi, Chemical hyperalgebra: dismutation reactions, *MATCH Communications in Mathematical and in Computer Chemistry* **67** (2012) 55–63.
8. B. Davvaz, A. Dehghan-Nezad and A. Benvidi, Chain reactions as experimental examples of ternary algebraic hyperstructures, *MATCH Communications in Mathematical and in Computer Chemistry* **65** (2) (2011) 491–499.
9. B. Davvaz and A. Dehghan-Nezhad, Dismutation reactions as experimental verifications of ternary algebraic hyperstructures, *MATCH Communications in Mathematical and in Computer Chemistry* **68** (2012) 551–559.
10. B. Davvaz, A. Dehghan-Nezad and M. M. Heidari, Inheritance examples of algebraic hyperstructures, *Information Sciences* **224** (2013) 180–187.
11. A. Dehghan-Nezhad, S. M. Moosavi-Nejad, M. Nadjafikhah and B. Davvaz, A physical example of algebraic hyperstructures: Leptons, *Indian Journal of Physics* **86** (11) (2012) 1027–1032.
12. B. Davvaz, A. Dehghan-Nezad and M. Mazloum-Ardakani, Chemical hyperalgebra: Redox reactions, *MATCH Communications in Mathematical and in Computer Chemistry* **71** (2014) 323–331.
13. B. Davvaz, A. Dehghan-Nezad and M. Mazloum-Ardakani, Describing the algebraic hyperstructure of all elements in radiolytic processes in cement medium, *MATCH Communications in Mathematical and in Computer Chemistry* **72** (2) (2014) 375–388.

14. B. Davvaz and V. Leoreanu-Fotea, Hyperring Theory and Applications, *International Academic Press,* **115**, Palm Harber, USA, 2007.
15. B. Davvaz, R. M. Santilli and T. Vougiouklis, Studies of multivalued hyperstructures for the characterization of matter-antimatter systems and their extension, *Algebras Groups and Geometries*, **28** (3) (2011) 105–116.
16. B. Davvaz, R. M. Santilli and T. Vougiouklis, Multi-Valued Hypermathematics for Characterization of Matter and Antimatter Systems, *Journal of Computational Methods in Sciences and Engineering* (JCMSE), **13** (2013) 37–50.
17. B. Davvaz, R. M. Santilli and T. Vougiouklis, Mathematical prediction of Ying's twin universes, *American Journal of Modern Physics* **4** (3) (2015) 5–9.
18. M. Ghadiri, B. Davvaz and R. Nekouian, Hv-Semigroup structure on F_2-offspring of a gene pool, *International Journal of Biomathematics* **5** (4) (2012) 1250011 (13 pages).
19. C. Harding, D.A. Johnson, R. Janes, Elements of the P Block, Published by Royal Society of Chemistry, 2002.
20. Š. Hošková, J. Chvalina and P. Račková, Transposition hypergroups of Fredholm integral operators and related hyperstructures. *Part I*, *Journal of Basic Science,* **4** (2008) 43–54.
21. Š. Hošková, J. Chvalina and P. Račková, Transposition hypergroups of Fredholm integral operators and related hyperstructures. *Part II*, G. *Journal of Basic Science* **4** (2008) 55–60.
22. R.T. Morrison and R. N. Boyd, Organic Chemistry, Sixth Eddition, Prentice-Hall Inc., 1992.
23. Mortimer, C. E., Chemistry: A Conceptual Approach, 4th ed., D. Van Nostrad Co., New York, 1979.
24. R.M. Santilli and T. Vougiouklis, Isotopies, genotopies, hyperstructures and their applications, New frontiers in hyperstructures, *New frontiers in hyperstructures* (Molise, 1995), 1–48, Ser. New Front. Adv. Math. Ist. Ric. Base, Hadronic Press, Palm Harbor, FL, 1996.
25. M.S. Sethi and M. Satake, Chemistry of Transition Elements, *South Asian Publishers*, New Delhi, 1988.
26. D.A. Skoog D.M. West and F.J. Holler, Saunders College Publishing, New York, 1992.
27. International Union of Pure and Applied Chemistry. "disproportionation". Compendium of Chemical Terminology Internet edition, http://en.wikipedia.org/wiki/Disproportionation.

28. T. Vougiouklis, The fundamental relation in hyperrings. The general hyperfield, *Algebraic Hyperstructures and Applications* (Xanthi, 1990), 203–211, World Sci. Publishing, Teaneck, NJ, 1991.
29. T. Vougiouklis, A new class of hyperstructures, *Journal of Combinatorics, Information & System Sciences* **20** (1995) 229–235.
30. T. Vougiouklis, H_v-groups defined on the same set, *Discrete Mathematics* **155** (1996) 259–265.
31. T. Vougiouklis, Convolutions on WASS hyperstructures, Combinatorics (Romeand Montesilvano, 1994). *Discrete Mathematics* **174** (1997) 347–355.
32. T. Vougiouklis, Hyperstructures and their Representations, Hadronic Press Inc., **115**, Palm Harber, 1994.