Computing GA_4 Index of Some Graph Operations

MAHBOOBEH SAHELI AND MARYAM JALALI RAD•

Institute of Nanoscience and Nanotechnology, University of Kashan, Kashan 87317-51167, I. R. Iran
Department of Mathematics, Faculty of Mathematical Sciences, University of Kashan, Kashan 87317 – 51167, I. R. Iran

(*Received March 7, 2011*)

ABSTRACT

The geometric-arithmetic index is another topological index was defined as $GA(G)=\sum_{uv\in E}\frac{2\sqrt{\deg_G(u)\deg_G(v)}}{\deg_G(u)+\deg_G(v)}$, in which degree of vertex u denoted by $\deg_G(u)$. We now define a new version of GA index as $GA_4(G)=\sum_{e=uv\in E(G)}\frac{2\sqrt{\varepsilon_G(u)\varepsilon_G(v)}}{\varepsilon_G(u)+\varepsilon_G(v)}$, where $\varepsilon_G(u)$ is the eccentricity of vertex u. In this paper we compute this new topological index for two graph operations.

Keywords: Topological index, GA Index, GA_4 index, graph operations.

1. INTRODUCTION

By a graph means a collection of points and lines connecting a subset of them. The points and lines of a graph also called vertices and edges of the graph, respectively. If e is an edge of G, connecting the vertices u and v, then we write $e = uv$ and say "u and v are adjacent". A connected graph is a graph such that there is a path between all pairs of vertices. The fact that many interesting graphs are composed of simpler graphs that serve as their basic building blocks prompts and justifies interest in the type of relationship that exist between various graph-theoretical invariants of composite graphs and of their components. The composite graphs considered here arise from simpler graphs via several binary operations. Such operations are sometimes called graph products, and the resulting graphs are also known as product graphs.

• Corresponding author.(e-mail: *jalali6834@gmail.com*)

Let G be a graph on n vertices. We denote the vertex and the edge set of G by $V(G)$ and $E(G)$, respectively. For two vertices u and v of $V(G)$ we define their distance $d_G(u, v)$ as the length of a shortest path connecting u and v in G. For a given vertex u of V(G) its eccentricity $\varepsilon_G(u)$ is the largest distance between u and any other vertex v of G. Hence, $\varepsilon_G(u) = \max_{v\in V(G)} d_G(u,v)$ [1-7]. The minimum and maximum eccentricity over all vertices of G are called the radius and diameter of G and denoted by $R(G)$ and $D(G)$, respectively.

The Zagreb indices have been introduced more than thirty years ago by Gutman and Trinajestić [8]. They are defined as:

$$M_1(G) = \sum_{v\in V(G)} \left(\deg_G(v)\right)^2 \text{ and } M_{2(}G) = \sum_{uv\in E(G)} \deg_G(u)\deg_G(v).$$

Now we define a new version of Zagreb indices as follows [9]:

$$M_1^*(G) = \sum_{uv\in E(G)} \varepsilon(u)+\varepsilon(v) \text{ and } M_2^*(G) = \sum_{uv\in E(G)} \varepsilon(u)\varepsilon(v).$$

It is easy to see that for every connected graph G, $M_2^*(G) = \xi(G)$.

A class of geometric–arithmetic topological indices may be defined as $GA_{general} = \sum_{uv\in E} \frac{2\sqrt{Q_uQ_v}}{Q_u+Q_v}$, where Q_u is some quantity that in a unique manner can be associated with the vertex u of the graph G, see [10]. The first member of this class was considered by Vukicević and Furtula [11], by setting Q_u to be the

$$GA(G) = \sum_{uv\in E} \frac{2\sqrt{\deg_G(u)\deg_G(v)}}{\deg_G(u)+\deg_G(v)},$$

where degree of vertex u denoted by $\deg_G(u)$. The second member of this class was considered by Fath-Tabar et al. [12] by setting Q_u to be the number $n_u = n_u(e|G)$ of vertices of G lying closer to the vertex u than to the vertex v for the edge uv of the graph G:

$$GA_2(G) = \sum_{uv\in E} \frac{2\sqrt{n_un_v}}{n_u+n_v}.$$

The third member of this class was considered by Zhou et al. [13] by setting Q_u to be the number $m_u = m_u(e|G)$ of edges of G lying closer to the vertex u than to the vertex v for the edge uv of the graph G:

$$GA_3(G) = \sum_{uv\in E} \frac{2\sqrt{m_um_v}}{m_u+m_v}.$$

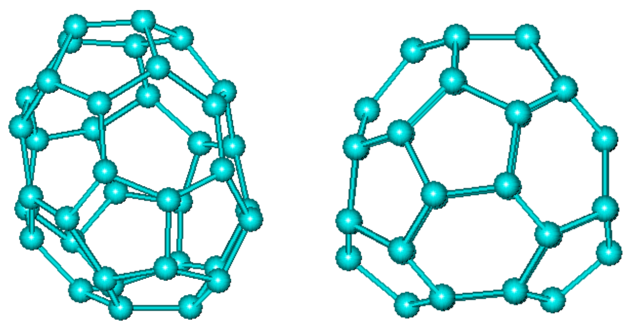
The fourth member of this class was defined by Ashrafi et al. [14] as follows:

$$GA_4(G) = \sum_{uv\in E} \frac{2\sqrt{\varepsilon_G(u)\varepsilon_G(v)}}{\varepsilon_G(u)+\varepsilon_G(v)},$$

where $\varepsilon_G(u)$ denotes to the eccentricity of vertex *u*.

A fullerene graph is a cubic 3-connected plane graph with (exactly 12) pentagonal faces and hexagonal faces. Let F_n be a fullerene graph with *n* vertices. By the Euler formula one can see that F_n has 12 pentagonal and $n/2 - 10$ hexagonal faces [15,16].

Sometimes GA_4 is a better descriptor for molecular structures than *GA* index. For example, consider two distinct isomers of fullerene C_{30} depicted in Figure 1. Since every fullerene graph is 3 regular, then $GA(C_{38}:1) = GA(C_{38}:2)$. But they have different GA_4 value. In other words, $GA_4(C_{38}:1) = 6$ and $GA_4(C_{38}:2) = 8$.



$\boldsymbol{C}_{38}$:1 $\boldsymbol{C}_{38}$:2

Figure 1. Two distinct isomers of C_{38}.

Throughout this paper our notation is standard and mainly taken from standard books of graph theory such as [17, 18] and [19 – 21]. All graphs considered in this paper are simple and connected.

2. MAIN RESULTS AND DISCUSSION

The aim of this section is to compute $GA_4(G)$, for some graph operations. Before going to calculate this index for graph operations, we must compute $GA_4(G)$, for some well-known class of graphs.

Example 1. Let K_n denotes the complete graph on *n* vertices. Then for every $v\in V(K_n)$, $\deg_G(v) = n-1$ and $\varepsilon_G(v) = 1$. This implies $GA_4(K_n) = \sum_{uv\in E(G)} \frac{2\sqrt{1}}{2} = \frac{n(n-1)}{2}$.

Example 2. Let C_n denotes the cycle of length n. If n is even then for every i, then i-th row of distance matrix of C_n is $1,2,...,\overset{i}{0},...,\ (n-1)/2,\ n/2,\ (n-1)/2,...,2,1$. When n is odd then the it is equal to $1,2,...,\overset{i}{0}\ ,...,\ (n-1)/2,\ (n-1)/2,...,2,1.$ Hence,

$$GA_4(C_n)=\begin{cases}\sum_{uv\in E(G)}\dfrac{2\sqrt{\frac{n}{2}\cdot\frac{n}{2}}}{\frac{n}{2}+\frac{n}{2}}=n & 2\mid n\\ \sum_{uv\in E(G)}\dfrac{2\sqrt{\frac{n-1}{2}\cdot\frac{n-1}{2}}}{\frac{n-1}{2}+\frac{n-1}{2}}=n & 2\nmid n\end{cases}.$$

Example 3. Let S_n be the star graph with n + 1 vertices, Figure 2. The central vertex is denoted by x and others vertices by u_1, u_2, …, u_n. Then for every $1 \le i , j \le n$, we have $d_G(x , u_i) = 1$ and $d_G (u_i , u_j) = 2$. So, $GA_4(S_n)=\sum_{uv\in E(G)}\frac{2\sqrt{2}}{3}=\frac{2\sqrt{2}}{3}n$.

Example 4. A wheel W_n is a graph of order n which contains a cycle of order n, and for which every vertex in the cycle is connected to other graph vertices, Figure 3. Suppose the central vertex is denoted by x and the others by u_1,u_2,,…,u_n. Then for every $1 \le i, j \le n$ we have $d_G (x , u_i) = 1$, $d_G (u_i , u_{i-1}) = 1$, $d_G (u_i , u_{i+1}) = 1$ and $d_G(u_i , u_j) = 2j(j \ne i - 1, i+1)$. So, $GA_4(W_n)=\frac{2\sqrt{2}}{3}n+n=(\frac{2\sqrt{2}}{3}+1)n$.

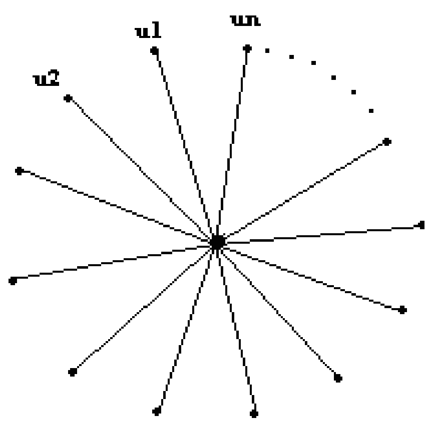


Figure 2. The Star Graph with n+1 Vertices.

Theorem 1.

$$GA_4(G) \geq \frac{2\sqrt{M_2^*(G)}}{M_1^*(G)}.$$

Proof.

$$[GA_4(G)]^2 = \sum_{uv\in E} \frac{4\varepsilon(u)\varepsilon(v)}{(\varepsilon(u)+\varepsilon(v))^2} + 4\sum_{uv\neq u'v'} \frac{\sqrt{\varepsilon(u)\varepsilon(v)}\sqrt{\varepsilon(u')\varepsilon(v')}}{(\varepsilon(u)+\varepsilon(v))(\varepsilon(u')+\varepsilon(v'))}$$

$$\geq \sum_{uv\in E} \frac{4\varepsilon(u)\varepsilon(v)}{(\varepsilon(u)+\varepsilon(v))^2} \geq \frac{M_2^*(G)}{[M_1^*(G)]^2}.$$

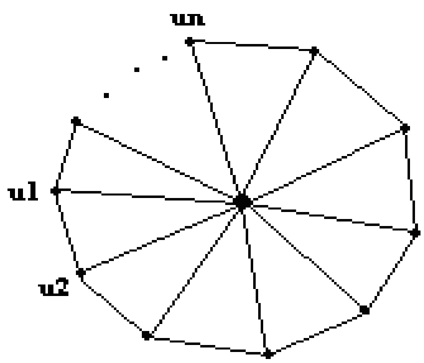


Figure 3. The Wheel Graph with n+1 Vertices.

Theorem 2. Let G be a graph with $m \geq 2$ edges. Then

$$\frac{2M_2^*(G)}{M_1^*(G)} \leq GA_4(G) \leq \frac{2}{3} M_2^*(G).$$

Proof. We can suppose $\varepsilon(u) = \varepsilon_G(u)$ for the vertex u in G. It is easy to see that for every e = uv in $E(G)$, $\varepsilon(u)+\varepsilon(v) \geq 3$. By the definition of GA_4 index we have

$$GA_4(G) = \sum_{uv\in E} \frac{2\sqrt{\varepsilon(u)\varepsilon(v)}}{\varepsilon(u)+\varepsilon(v)} \leq \frac{2}{3}\sum_{uv\in E} \sqrt{\varepsilon(u)\varepsilon(v)}$$

$$\leq \frac{2}{3}\sum_{uv\in E} \varepsilon(u)\varepsilon(v) = \frac{2}{3} M_2^*(G).$$

On the other hand,

$$GA_4(G) = \sum_{uv\in E} \frac{2\sqrt{\varepsilon(u)\varepsilon(v)}}{\varepsilon(u)+\varepsilon(v)} \geq 2\frac{\sum_{uv\in E}\sqrt{\varepsilon(u)\varepsilon(v)}}{M_1^*(G)} = \frac{2M_2^*(G)}{M_1^*(G)}.$$

This completes the proof.

The ***join*** $G = G_1 + G_2$ of graphs G_1 and G_2 with disjoint vertex sets V_1 and V_2 and edge sets E_1 and E_2 is the graph union $G_1 \cup G_2$ together with all the edges joining V_1 and V_2. It is easy to see that $|V(G_1+G_2)| = n_1 n_2$ and $|E(G_1+G_2)| = m_1 + m_2 + n_1 n_2$.

Lemma 3 [19].

$$\varepsilon_{G_1+G_2}(u) = \begin{cases} 1 & \varepsilon_{G_1}(u) = 1 \quad or \quad \varepsilon_{G_2}(u) = 1 \\ 2 & \varepsilon_{G_1}(u) \geq 2 \quad or \quad \varepsilon_{G_2}(u) \geq 2 \end{cases}.$$

Theorem 4. Let G_1 and G_2 be connected graphs, where $w_i = |\{u \in V(G_i), \varepsilon_{G_i}(u) = 1\}|$ for i= 1,2.

$$GA_4(G_1+G_2) = m_1 + m_2 + n_1 n_2 + \frac{2\sqrt{2}}{3}(\omega_1 + \omega_2)(n_1 + n_2 + \omega_1 + \omega_2).$$

Proof. Let $\varepsilon(u) = \varepsilon_{G_1+G_2}(u)$. So, we have

$$GA_4(G_1+G_2) = \sum_{uv \in E(G_1+G_2)} \frac{2\sqrt{\varepsilon(u)\cdot\varepsilon(v)}}{\varepsilon(u)+\varepsilon(v)}$$

$$= \sum_{\substack{uv \in E(G_1+G_2),\\ uv \in E(G_1)}} \frac{2\sqrt{\varepsilon(u)\cdot\varepsilon(v)}}{\varepsilon(u)+\varepsilon(v)} + \sum_{\substack{uv \in E(G_1+G_2),\\ uv \in E(G_2)}} \frac{2\sqrt{\varepsilon(u)\cdot\varepsilon(v)}}{\varepsilon(u)+\varepsilon(v)} + \sum_{\substack{uv \in E(G_1+G_2),\\ uv \notin E(G_1), uv \notin E(G_2)}} \frac{2\sqrt{\varepsilon(u)\cdot\varepsilon(v)}}{\varepsilon(u)+\varepsilon(v)}.$$

By using table 1, it is easy to see that:

$$\sum_{\substack{uv \in E(G_1+G_2),\\ uv \in E(G_1)}} \frac{2\sqrt{\varepsilon(u)\cdot\varepsilon(v)}}{\varepsilon(u)+\varepsilon(v)} = \sum_{\varepsilon(u)=\varepsilon(v)=1} 1 + \sum_{\substack{\varepsilon(u)=1,\\ \varepsilon(v)=2}} \frac{2\sqrt{2}}{3} + \sum_{\varepsilon(u)=\varepsilon(v)=2} 1$$

$$= \binom{w_1}{2} + \frac{2\sqrt{2}}{3} w_1 \times (n_1 - w_1) + \left(m_1 - \binom{w_1}{2} - w_1 \times (n_1 - w_1) \right)$$

$$= m_1 + \frac{2\sqrt{2}}{3}\left(w_1 \times (n_1 - w_1)\right),$$

$$\sum_{\substack{uv \in E(G_1+G_2),\\ uv \in E(G_1)}} \frac{2\sqrt{\varepsilon(u)\cdot\varepsilon(v)}}{\varepsilon(u)+\varepsilon(v)} = \sum_{\varepsilon(u)=\varepsilon(v)=1} 1 + \sum_{\substack{\varepsilon(u)=1,\\ \varepsilon(v)=2}} \frac{2\sqrt{2}}{3} + \sum_{\varepsilon(u)=\varepsilon(v)=2} 1$$

$$= \binom{w_2}{2} + \frac{2\sqrt{2}}{3} w_2 \times (n_2 - w_2) + \left(m_2 - \binom{w_2}{2} - w_2 \times (n_2 - w_2) \right)$$

$$= m_2 + \frac{2\sqrt{2}}{3}\left(w_2 \times (n_2 - w_2)\right).$$

For computing the term $\sum_{\substack{uv\in E(G_1+G_2),\\ uv\notin E(G_1), uv\notin E(G_2)}} \frac{2\sqrt{\varepsilon(u)\cdot\varepsilon(v)}}{\varepsilon(u)+\varepsilon(v)}$, according to table 1 we should to consider the following classes of edges:

Case 1: Number of edges with $\varepsilon_{G_1}(u) = \varepsilon_{G_2}(v) = 1$ is $w_1 \times w_2$, where $uv \in E(G_1 + G_2), uv \notin E(G_1), uv \notin E(G_2)$.

Case 2: Number of edges with $\varepsilon_{G_1}(u) = 1$ and $\varepsilon_{G_2}(v) = 2$ is

$$(n_1 - w_1) \times w_2 + (n_2 - w_2) \times w_1 .$$

Case 3: Number of edges with $\varepsilon_{G_1}(u) = \varepsilon_{G_2}(v) = 2$ is $(n_1 - w_1) \times (n_2 - w_2)$.

Therefore,

$$\sum_{\substack{uv\in E(G_1+G_2),\\ uv\notin E(G_1), uv\notin E(G_1)}} \frac{2\sqrt{\varepsilon(u)\cdot\varepsilon(v)}}{\varepsilon(u)+\varepsilon(v)} = \sum_{\varepsilon_{G_1}(u)=\varepsilon_{G_2}(v)=1} 1 + \sum_{\substack{\varepsilon_{G_1}(u)=1,\\ \varepsilon_{G_2}(v)=2}} \frac{2\sqrt{2}}{3} + \sum_{\varepsilon_{G_1}(u)=\varepsilon_{G_2}(v)=2} 1$$

$$= w_1 w_2 + \frac{2\sqrt{2}}{3}\left((n_1 - w_1)w_2 + (n_2 - w_2)w_1\right) + (n_1 - w_1)(n_2 - w_2).$$

Finally, we have:

$$GA_4(G_1 + G_2) = m_1 + m_2 + n_1 n_2 + \frac{2\sqrt{2}-3}{3}(n_1 + n_2)(w_1 + w_2) - \frac{2\sqrt{2}-3}{3}(w_1 + w_2)^2.$$

Corollary 5. If $w_1 = w_2 = 0$, then $GA_4(G_1 + G_2) = m_1 + m_2 + n_1 n_2 = |E(G_1 + G_2)|$.

Lemma 6 [18]. Let $G_1, \ldots, G_k$ be some connected graphs. Then:

1) $$|E(G_1 + \cdots + G_k)| = \sum_{i=1}^{k} |E(G_i)| + \frac{1}{2}\sum_{i=1}^{k} |V(G_i)| \sum_{\substack{j=1\\ j\neq i}}^{k} |V(G_j)|$$

$$= \sum_{i=1}^{k} m_i + \frac{1}{2}\sum_{i=1}^{k} n_i \sum_{\substack{j=1\\ j\neq i}}^{k} n_j ,$$

2) $$\varepsilon_{(G_1+\cdots+G_k)}(u) = \begin{cases} 1 & \exists i: \quad \varepsilon_{G_i}(u) = 1 \\ 2 & \exists i: \quad \varepsilon_{G_i}(u) \geq 2 \end{cases}.$$

Table 1. Values of $\varepsilon_G(u), \varepsilon_G(v)$ for edges $e = uv$.

Graph	G_1	G_2
#Vertices	n_1	n_2
#Edges	m_1	m_2
#Edges with $\varepsilon_G(u)=\varepsilon_G(v)=1$	$\binom{w_1}{2}$	$\binom{w_2}{2}$
# Edges with $\varepsilon_G(u)=1, \varepsilon_G(v)\geq 2$	$w_1\times(n_1-w_1)$	$w_2\times(n_2-w_2)$
#Edges with $\varepsilon_G(u)\geq 2, \varepsilon_G(v)\geq 2$	$m_1-\binom{w_1}{2}-w_1\times(n_1-w_1)$	$m_2-\binom{w_2}{2}-w_2\times(n_2-w_2)$
# Vertices with $\varepsilon_G(u)=1$	w_1	w_2

Theorem 7. Let $G_1, \ldots, G_k$ be some connected graphs. Then:

$$GA_4(G_1+\cdots+G_k)=m+\frac{2\sqrt{2}-3}{3}+\sum_{i=1}^{k}n_i\times\sum_{i=1}^{k}w_i-\frac{2\sqrt{2}-3}{3}(\sum_{i=1}^{k}w_i)^2.$$

Corollary 8. If $\sum_{i=1}^{k}w_i=0$, then $GA_4(G_1+\cdots+G_k)=|E(G_1+\cdots+G_k)|=m$.

The ***disjunction*** $G_1\vee G_2$ of graphs G_1 and G_2 is the graph with vertex set $V(G_1)\times V(G_2)$ and (u_1,v_1) is adjacent with (u_2,v_2) whenever $u_1u_2\in E(G_1)$ or $v_1v_2\in E(G_2)$. Further, $|V(G_1\vee G_2)|=n_1n_2$ and $|E(G_1\vee G_2)|=m_1n_2^2+m_2n_1^2-2m_1m_2$.

Lemma 9 [19].

$$\varepsilon_{G_1\vee G_2}(a,x)=\begin{cases}1 & \varepsilon_{G_1}(a)=1 \quad and \quad \varepsilon_{G_2}(x)=1\\ 2 & \varepsilon_{G_1}(a)\geq 2 \quad or \quad \varepsilon_{G_2}(x)\geq 2\end{cases}.$$

Theorem 10.

$$GA_4(G_1\vee G_2)=m+\frac{2\sqrt{2}-3}{3}\left(w_1w_2\times(n_1n_2-w_1w_2)\right).$$

Proof.

$$GA_4(G_1\vee G_2)=\sum_{(a,x)(b,y)\in E(G_1\vee G_2)}\frac{2\sqrt{\varepsilon(a,x)\cdot\varepsilon(b,y)}}{\varepsilon(a,x)+\varepsilon(b,y)}=\sum_{\substack{\varepsilon(a,x)=1,\\ \varepsilon(b,y)=1}}1+\sum_{\substack{\varepsilon(a,x)=1,\\ \varepsilon(b,y)=2}}\frac{2\sqrt{2}}{3}+\sum_{\substack{\varepsilon(a,x)=2,\\ \varepsilon(b,y)=2}}1 .$$

The number of edges $e = uv$ with $\varepsilon(u)=\varepsilon(v)=1$ is $\binom{w_1w_2}{2}$. Similarly, it is easy to see the number of edges $e = uv$ with $\varepsilon(u)=1$ and $\varepsilon(v)=2$ is $w_1w_2\times(n_1n_2-1-(w_1w_2-1)) = w_1w_2\times(n_1n_2-w_1w_2)$. Finally, the number of edges $e = uv$ with $\varepsilon(u)=\varepsilon(v)=2$ is $m-\binom{w_1w_2}{2}-w_1w_2\times(n_1n_2-w_1w_2)$.

Corollary 11. If $w_1=0$ or $w_2=0$, then $GA_4(G_1\vee G_2)=m=|E(G_1\vee G_2)|$.

The ***symmetric difference*** $G_1\oplus G_2$ of two graphs G_1 and G_2 is the graph with vertex set $V(G_1)\times V(G_2)$ and $E(G_1\oplus G_2)=\{(u_1,u_2)(v_1,v_2)\,|\,u_1v_1\in E(G_1) \text{ or } u_2v_2\in E(G_2)\}$. Also, $|V(G_1\oplus G_2)|=n_1n_2$ and $|E(G_1\oplus G_2)|=m_1n_2^2+m_2n_1^2-4m_1m_2$.

Lemma 12 [19]. $\varepsilon_{G_1\oplus G_2}(a,x)=2$.

Theorem 13. $GA_4(G_1\oplus G_2)=|E(G_1\oplus G_2)|$**.**

Proof. $GA_4(G_1\oplus G_2)=\sum_{(a,x)(b,y)\in E(G_1\oplus G_2)}\frac{2\sqrt{\varepsilon(a,x)\cdot\varepsilon(b,y)}}{\varepsilon(a,x)+\varepsilon(b,y)}=\sum_{(a,x)(b,y)\in E(G_1\oplus G_2)}1.$

REFERENCES

1. V. Sharma, R. Goswami and A. K. Madan, Eccentric connectivity index: a novel highly discriminating topological descriptor for structure-property and structure-activity studies, *J. Chem. Inf. Comput. Sci*. **37** (1997) 273 –282.
2. H. Dureja and A. K. Madan, Superaugmented eccentric connectivity indices: new-generation highly discriminating topological descriptors for QSAR/QSPR modeling, *Med. Chem. Res*. **16** (2007) 331–341.
3. V. Kumar, S. Sardana and A. K. Madan, Predicting anti-HIV activity of 2,3-diary l-1,3-thiazolidin-4-ones: computational approaches using reformed eccentric connectivity index, *J. Mol. Model*. **10** (2004) 399–407.
4. M. Fischermann, A. Homann, D. Rautenbach, L. A. Szekely and L. Volkmann, Wiener index versus maximum degree in trees, *Discrete Appl. Math.* **122** (2002) 127–137.

5. S. Gupta, M. Singh and A. K. Madan, Application of graph theory: relationship of eccentric connectivity index and wiener's index with anti-inflammatory activity, *J. Math. Anal. Appl*. **266** (2002) 259–268.
6. S. Sardana and A. K. Madan, Application of graph theory: relationship of molecular connectivity index, Wiener's index and eccentric connectivity index with diuretic activity, *MATCH Commun. Math. Comput. Chem*. **43** (2001) 85–98.
7. R. Todeschini and V. Consonni, Handbook of Molecular Descriptors, Wiley-VCH, Weinheim, 2000.
8. N. Trinajstić and I. Gutman, *Mathematical Chemistry*, *Croat. Chem. Acta* **75** (2002) 329 – 356.
9. M. Ghorbani and M. A. Hosseinzadeh, A New Version of Zagreb Indices, *Filomat* **26(1)** (2012) 93 – 100.
10. B. Furtula, A. Graovac and D. Vukičević, Atom–bond connectivity index of trees, *Disc. Appl. Math.* **157** (2009) 2828 – 2835.
11. D. Vukičević and B. Furtula, Topological index based on the ratios of geometrical and arithmetical means of end-vertex degrees of edges, *J. Math. Chem*. **46** (2009) 1369 − 1376.
12. G. H. Fath-Tabar, B. Furtula and I. Gutman, A new geometric--arithmetic index, *J. Math. Chem*. **47** (2010) 477 – 486.
13. B. Zhou, I. Gutman, B. Furtula and Z. Du, On two types of geometric-arithmetic index, *Chem. Phys. Lett*. **482** (2009) 153 – 155.
14. M. Ghorbani and A. Khaki, A note on the fourth version of geometric-arithmetic index, Optoelectron. *Adv. Mater. – Rapid Comm*. **4(12)** (2010) 2212–2215.
15. H. W. Kroto, J. R. Heath, S. C. O'Brien, R. F. Curl and R. E. Smalley, C_{60}: Buckminsterfullerene, *Nature* **318** (1985) 162–163.
16. H. W. Kroto, J. E. Fichier and D. E Cox, *The Fullerene*, Pergamon Press, New York, 1993.
17. I. Gutman and N. Trinajstić, Graph theory and molecular orbitals. Total φ-electron energy of alternant hydrocarbons, *Chem. Phys. Lett*. **17** (1972) 535–538
18. D. B. West, *Introduction to Graph theory,* Prentice Hall, Upper Saddle River, 1996.
19. T. Doslic, M. Ghorbani and M. A. Hosseinzadeh, Eccentric connectivity polynomial of some graph operations, *Util. Math.* **84** (2011) 297–309.
20. M. Ghorbani, GA index of TUC_4C_8(R) nanotube, *Optoelectronics and Advanced Materials*-(*RC*) **4(2)** (2010) 261 – 263.
21. A. R. Ashrafi, M. Saheli and M. Ghorbani, The eccentric connectivity index of $TUC_4C_8(R)$ nanotubes, *J. Comput. Appl. Math.* **235** (2011) 4561 – 4566.