On the edge reverse Wiener indices of $TUC_4C_8(S)$ nanotubes

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ABSTRACT

The edge versions of reverse Wiener indices were introduced by Mahmiani et al. very recently. In this paper, we find their relation with ordinary (vertex) Wiener index in some graphs. Also, we compute them for trees and TUC_4C_8(s) naotubes.

Keywords: Molecular graph, Molecular matrix, Reveres Wiener indices, Edge reverse Wiener indices, Distance of graph, Line graph, Nanotubes.

1 INTRODUCTION

The vertex-Wiener number is defined as the sum of all distances in the hydrogen-depleted graph [8]. The distance d_{ij} between two graph vertices v_i and v_j is the number of edges along the shortest path between these two vertices. The matrix which has as entries d_{ij} (topological distances) is called the distance matrix D of the graph. Then, the vertex-Wiener number for a graph G with n vertices, defined as:

$$W_v(G) = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} d_{ij}$$

The vertex reverse Wiener number introduced by Alexandru T. Balanan and et al. in [1]. They defined at first the distance matrix $[RD_v]$ as follow:

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The diameter $\Delta(G)$ of a graph is the largest distance between any two vertices (i.e. $\Delta(G) = \max\{d_{ij} | \ i,j \in V(G), i \neq j \ \}$). Starting from the distance matrix and subtracting from $\Delta(G)$ each d_{ij} value, one obtains a new symmetrical matrix which, like the distance matrix, has zeroes on the main diagonal and, in addition, at least a pair of zeroes off the main diagonal corresponding to the diameter in the distance matrixRD, then:

$$[RD_v]_{ij} = \begin{cases} \Delta(G) - d_{ij} & ,i \neq j \\ 0 & ,i = j \end{cases}$$

where d_{ij} is theij-th element of the distance matrix$D$which is equal tothe graph distance between vertices v_i and v_j on the shortest path between them. Therefore, the reverse vertex-Wiener number for a graph G with n vertices, defined as:

$$RW_v = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}[RD_v]_{ij}$$

The vertex complementary distance matrix $CD_v = CD_v(G)$ of a graph G with n vertices is the square $n \times n$ symmetric matrix whose elements are defined as [5]:

$$[CD_v]_{ij} = \begin{cases} \Delta(G) + 1 - d_{ij} & ,i \neq j \\ 0 & ,i = j \end{cases}$$

It can be observed that all entries in the reverse vertex-Wiener matrix RD_v are lower by 1 than those in the vertex complementary distance matrix CD_v.

In 2009, Iranmanesh et. al. introduced the edge versions of Wiener index as follows. At first, they introduced the distances between edges [3].

Let $e = (u,v)$, $f = (x,y) \in E(G)$ and d be the distance between vertices on shortest path. The distances between each two edges $e = xy$ and $f = uv$ are

$$d_0(e,f) = \begin{cases} d_1(e,f) + 1 & ,e \neq f \\ 0 & ,e = f \end{cases} \text{ and } d_4(e,f) = \begin{cases} d_2(e,f) & ,e \neq f \\ 0 & ,e = f \end{cases}$$

Where $d_1(e,f) = min\{d(x,u), d(x,v), d(y,u), d(y,v)\}$ and $d_2(e,f) = max\{d(x,u), d(x,v), d(y,u), d(y,v)\}$.

Then, the edge-Wiener numbers introduced for a graph G in [3], according to distances d_k, $k = 0,4$, are:

$$W_{ek}(G) = \sum_{\{i,j\} \subseteq E(G)} d_k(e,f) .$$

Recently, Mahmiani et al. introduced the edge versions of reverse Wiener indices in [6]. They firstly introduced the edge reverse Wiener matrix RD_k and edge complementary distance matrix CD_k ,$k = 0,4$which are:

$$[RD_k]_{ij} = \begin{cases} \Delta_k(G) - [D_k]_{ij} & ,i \neq j \\ 0 & ,i = j \end{cases} \text{ and } [CD_k]_{ij} = \begin{cases} \Delta_k(G) + 1 - [D_k]_{ij} & ,i \neq j \\ 0 & ,i = j \end{cases}$$

where D_k, $k = 0,4$, are the edge distance matrices which have entries $(d_k)_{ij}$, $[D_k]_{ij}$ is the ij -th element of the edge distance matrix D_k which is equal to distance d_k between edges e_i and e_j, and $\Delta_k(G) = max\{(d_k)_{ij}|j,\ i\in E(G),\ i\neq j\}$. In follows, for convenience, we use the notation $RD_k(i,j)$ and $CD_k(i,j)$ instead of $[RD_k]_{ij}$ and $[CD_k]_{ij}$, respectively.

Then, they introduced the reverse edge-Wiener index RW_{ek} and the edge complementary Wiener index CW_{ek} , $k = 0,4$, by using these matrices as follows :

$$RW_{ek} = \sum_{\{i,j\}\subseteq E(G)} RD_k(i,j) \text{ and } CW_{ek} = \sum_{\{i,j\}\subseteq E(G)} CD_k(i,j)\,.$$

Some relations among between then are concluded in [6].

Theorem 1-1.[6] The relation among the reverse edge-Wiener index RW_{ek}, the edge complementary Wiener index CW_{ek} and edge Wiener indices W_{ek} , $k = 0,4$, are:

1. $RW_{ek} + W_{ek} = \Delta_k(G)\binom{m}{2}$

2. $CW_{ek} + W_{ek} = (\Delta_k(G)+1)\binom{m}{2}$

3. $CW_{ek} - RW_{ek} = \binom{m}{2}$ ■

In following, we find the relation of edge versions of reverse Wiener indices with vertex Wiener index in some graphs. Also, we compute them for trees and $TUC_4C_8(S)$ nanotubes.

2 COMPUTATIONS

Firstly we state the relations among the reverse edge-Wiener index, the edge complementary Wiener index and vertex Wiener indices in graphs which vertices has degree 2 and 3.

Before stating these relations, we restate the edge Wiener indices according to distances between vertices which are found by Iranmanesh et al in [4].

Theorem 2-1. [4] The edge Wiener indices according to the distances between vertices are

1. $W_{e0}(G)=\frac{1}{8}\sum_{x\in V(G)}\sum_{y\in V(G)}\deg(x)\times\deg(y)\times d(x,y)-\frac{m}{4}+\sum_{\{e,f\}\in A_3}\left(\frac{1}{2}\right)+\sum_{\{e,f\}\in A_2}\left(\frac{1}{4}\right)+\sum_{\{e,f\}\in A_4}\left(\frac{3}{4}\right)+|C|.$

2. $W_{e4}(G)=\frac{1}{8}\sum_{x\in V(G)}\sum_{y\in V(G)}\deg(x)\times\deg(y)\times d(x,y)-\frac{m}{4}+\sum_{\{e,f\}\in A_3}\left(\frac{1}{2}\right)+\sum_{\{e,f\}\in A_2}\left(\frac{1}{4}\right)+\sum_{\{e,f\}\in A_4}\left(\frac{3}{4}\right)+|A_1|.$

3. $W_{e4}(G)=W_{e0}(G)+|A_1|-|C|.$

where $A_1=\{\{e,f\}\subseteq E(G)\mid d_3(e,f)=d'(e,f)\}$, $A_2=\left\{\{e,f\}\subseteq E(G)\,\middle|\, d_3(e,f)=d'(e,f)+\frac{1}{4}\right\}$,
$A_3=\left\{\{e,f\}\subseteq E(G)\,\middle|\, d_3(e,f)=d'(e,f)+\frac{2}{4}\right\}$, $A_4=\left\{\{e,f\}\subseteq E(G)\,\middle|\, d_3(e,f)=d'(e,f)+\frac{3}{4}\right\}$
and $C=\left\{\{e,f\}\subseteq E(G)\,\middle|\,\begin{array}{l} if\ e=uv\ and\ f=xy\ ;\\ d(u,x)=d(u,y)=d(v,x)=d(v,x)\end{array}\right\}.$ ■

Now, we are ready to state our desire relations.

Theorem 2-2. The relation among the reverse edge-Wiener index, the edge complementary Wiener index and vertex Wiener indices in graphs which vertices has degree 2 and 3 are

1. $$RW_{e0}=\Delta_0(G)\binom{m}{2}-\frac{9}{4}W_v(G)-\frac{3}{8}\sum_{\substack{x\in V(G)\\ \deg(x)=2}}\sum_{y\in V(G)}d(x,y)+\sum_{\substack{x\in V(G)\\ \deg(x)=2}}\sum_{\substack{y\in V(G)\\ \deg(y)=2}}d(x,y)+\frac{m}{4}-\sum_{\{e,f\}\in A_3}\left(\frac{1}{2}\right)-\sum_{\{e,f\}\in A_2}\left(\frac{1}{4}\right)-\sum_{\{e,f\}\in A_4}\left(\frac{3}{4}\right)-|C|$$

2. $$RW_{e4}=\Delta_4(G)\binom{m}{2}-\frac{9}{4}W_v(G)-\frac{3}{8}\sum_{\substack{x\in V(G)\\ \deg(x)=2}}\sum_{y\in V(G)}d(x,y)+\sum_{\substack{x\in V(G)\\ \deg(x)=2}}\sum_{\substack{y\in V(G)\\ \deg(y)=2}}d(x,y)+\frac{m}{4}-\sum_{\{e,f\}\in A_3}\left(\frac{1}{2}\right)-\sum_{\{e,f\}\in A_2}\left(\frac{1}{4}\right)-\sum_{\{e,f\}\in A_4}\left(\frac{3}{4}\right)-|A_1|$$

3. $$CW_{e0}=(\Delta_0(G)+1)\binom{m}{2}-\frac{9}{4}W_v(G)-\frac{3}{8}\sum_{\substack{x\in V(G)\\ \deg(x)=2}}\sum_{y\in V(G)}d(x,y)+\sum_{\substack{x\in V(G)\\ \deg(x)=2}}\sum_{\substack{y\in V(G)\\ \deg(y)=2}}d(x,y)+\frac{m}{4}-\sum_{\{e,f\}\in A_3}\left(\frac{1}{2}\right)-\sum_{\{e,f\}\in A_2}\left(\frac{1}{4}\right)-\sum_{\{e,f\}\in A_4}\left(\frac{3}{4}\right)-|C|$$

4. $$CW_{e4} = \left(\Delta_4(G)+1\right)\binom{m}{2} - \frac{9}{4}W_v(G) - \frac{3}{8}\sum_{\substack{x\in V(G)\\ \deg(x)=2}}\sum_{y\in V(G)} d(x,y) + \sum_{\substack{x\in V(G)\\ \deg(x)=2}}\sum_{\substack{y\in V(G)\\ \deg(y)=2}} d(x,y) +$$
$$\frac{m}{4} - \sum_{\{e,f\}\in A_3}\left(\frac{1}{2}\right) - \sum_{\{e,f\}\in A_2}\left(\frac{1}{4}\right) - \sum_{\{e,f\}\in A_4}\left(\frac{3}{4}\right) - |A_1|$$

Proof. According to Theorems (1-1 and 2-1) and degree of vertices which are 2 and 3, we can state these relations easily. ■

Now, we compute the edge versions of reverse Wiener indices for trees in terms of vertex Wiener index. Before stating this computation, we mention to edge Wiener indices of trees. The first edge Wiener index of trees is computed in [2].

Lemma 2-3. [2] Let *T* be a tree with *n* vertices. Then, the first edge-Wiener number of *T* is

$$W_{e0}(T) = W_v(T) - \binom{n}{2}. \quad ■$$

Also we have due to the relation between first and second edge Wiener numbers in Theorem (2-1).

Lemma 2-4. The relation between different versions of edge-Wiener numbers for a tree T with *n* vertices is

$$W_{e4}(T) = W_{e0}(T) - \binom{n-1}{2} \quad ■$$

Therefore, in the following theorem we state the edge reverse Wiener numbers and edge.

Theorem 2-5. The edge reverse Wiener numbers and edge complementary Wiener numbers of trees in terms of Wiener number are

1. $RW_{e0}(T) = \Delta_0(G)\binom{n-1}{2} - W_v(T) + \binom{n}{2}$,
2. $RW_{e4}(T) = \left(\Delta_4(G)+1\right)\binom{n-1}{2} - W_v(T) + \binom{n}{2}$,
3. $CW_{e0}(T) = (\Delta_0(G)+1)\binom{n-1}{2} - W_v(T) + \binom{n}{2}$,
4. $CW_{e4}(T) = \left(\Delta_4(G)+2\right)\binom{n-1}{2} - W_v(T) + \binom{n}{2}$,

where $k = 0, 4$.

Proof. Let T be a tree with n vertices. According to Theorems (3-1, 3-2 and 4-1) and Lemma (4-2), we can concluded the desire results as follows.

1. $RW_{e0}(T) = \Delta_0(G)\binom{n-1}{2} - W_{e0}(T) = \Delta_0(G)\binom{n-1}{2} - W_v(T) + \binom{n}{2}$

2. $$\begin{aligned} RW_{e4}(T) &= \Delta_4(G)\binom{n-1}{2} - W_{e4}(T) = \left(\Delta_4(G)+1\right)\binom{n-1}{2} - W_{e0}(T) \\ &= \left(\Delta_4(G)+1\right)\binom{n-1}{2} - W_v(T) + \binom{n}{2} \end{aligned}$$

3. $CW_{e0}(T) = (\Delta_0(G)+1)\binom{n-1}{2} - W_{e0}(T) = (\Delta_0(G)+1)\binom{n-1}{2} - W_v(T) + \binom{n}{2}$

4. $$\begin{aligned} CW_{e4}(T) &= (\Delta_4(G)+1)\binom{n-1}{2} - W_{e4}(T) = \left(\Delta_4(G)+2\right)\binom{n-1}{2} - W_{e0}(T) \\ &= \left(\Delta_4(G)+2\right)\binom{n-1}{2} - W_v(T) + \binom{n}{2} \end{aligned}$$ ■

In what follows, the second edge reverse Wiener index, RW_{e4}, and the edge complementary Wiener indices CD_{ek}, $k = 0,4,$ for $TUC_4C_8(S)$ nanotube are computed.

In $TUC_4C_8(S)$ nanotube, p is the number of squares in a row and q is the number of rows which is shown in Figure 1.

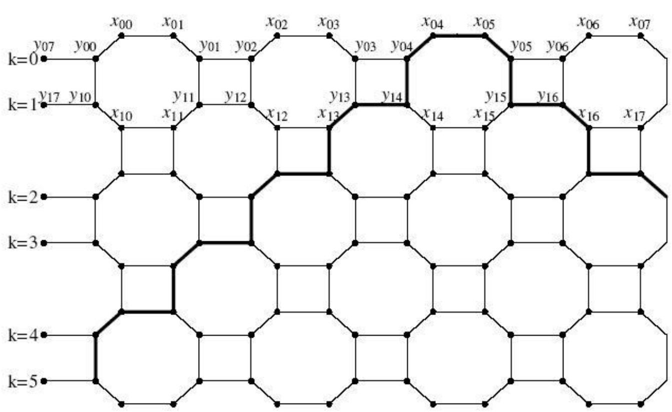

Figure 1. The TUC4C8(S) lattice with p = 4 and q = 6.

In [7], the first edge Wiener number of this nanotube is calculated.

Theorem 2-6. ([7]) The first version of edge-Wiener index of $TUC_4C_8(S)=T(p,q)$ can be computed by the following formulas:

1. If p is even, then:

$$W_{e0}(T(p,q))=\begin{cases}\frac{3}{2}pq^4+18p^3q^2+6p^2q^3-\frac{pq^2}{2}-8p^2q+6p^3q+3p^2q^2-6pq+pq^3- \\ \qquad q^2-2q-5p^2+2p+2+2\left[\frac{2p-2q+1}{4}\right] & ,q\le p \\ \frac{15}{2}p^5+6p^4q+\frac{3p^3}{2}+12p^2q^3-11p^2q+3p^3q+6p^2q^2-7pq-4p^2+ \\ \qquad 3p+pq^2+1 & ,q>p\end{cases}$$

2. If p is odd, then:

$$W_{e0}(T(p,q))=\begin{cases}\frac{3}{2}pq^4+18p^3q^2+6p^2q^3-\frac{pq^2}{2}-8p^2q+6p^3q+3p^2q^2-6pq+pq^3- \\ \qquad q^2-q-3p^2+2\left[\frac{2p-2q+1}{4}\right]-8\left[\frac{p}{2}\right]-8\left[\frac{p}{2}\right]^2 & ,q\le p \\ \frac{15}{2}p^5+6p^4q+\frac{3p^3}{2}+12p^2q^3-11p^2q+3p^3q+6p^2q^2-7pq-2p^2+ \\ \qquad 3p+pq^2-8\left[\frac{p}{2}\right]-8\left[\frac{p}{2}\right]^2 & ,q>p\end{cases}$$

■

In [6], the first edge reverse Wiener index, RW_{e0}, is computed as follows.

Theorem 2-7. [6] The first edge reverse Wiener number, RW_{e0}, of $TUC_4C_8(S)=T(p,q)$ nanotubes is

1. If p is even, then:

$$RW_{e0}(T(p,q))=\begin{cases}(2p+2)\binom{6pq-2p}{2}-\frac{3}{2}pq^4-18p^3q^2-6p^2q^3+\frac{pq^2}{2}+8p^2q-6p^3q- \\ \qquad 3p^2q^2+6pq-pq^3+q^2+2q+5p^2-2p-2-2\left[\frac{2p-2q+1}{4}\right] & ,q\le p \\ (2q+2)\binom{6pq-2p}{2}-\frac{15}{2}p^5-6p^4q-\frac{3p^3}{2}-12p^2q^3+11p^2q-3p^3q-6p^2q^2+ \\ \qquad 7pq+4p^2-3p-pq^2-1 & ,q>p\end{cases}$$

2. If p is odd, then:

$$RW_{e0}(T(p,q))=\begin{cases} (4\frac{p-1}{2}+2)\binom{6pq-2p}{2}-\frac{3}{2}pq^4-18p^3q^2-6p^2q^3+\frac{pq^2}{2}+8p^2q-6p^3q- \\ 3p^2q^2+6pq-pq^3+q^2+q+3p^2-2\left[\frac{2p-2q+1}{4}\right]+8\left[\frac{p}{2}\right]+8\left[\frac{p}{2}\right]^2 & ,q\le p \\ (2q)\binom{6pq-2p}{2}-\frac{15}{2}p^5-6p^4q-\frac{3p^3}{2}-12p^2q^3+11p^2q-3p^3q- \\ 6p^2q^2+7pq+2p^2-3p-pq^2+8\left[\frac{p}{2}\right]+8\left[\frac{p}{2}\right]^2 & ,q>p \end{cases}$$

Now, we compute the first edge complementary Wiener indices CW_{e0} of $TUC_4C_8(S)$.

Theorem 2-8. The first edge complementary Wiener indices CW_{e0} of $TUC_4C_8(S)=T(p,q)$ is

1. If p is even, then:

$$CW_{e0}(T(p,q))=\begin{cases} (2p+4)\binom{6pq-2p}{2}-\frac{3}{2}pq^4-18p^3q^2-6p^2q^3+\frac{pq^2}{2}+8p^2q-6p^3q- \\ 3p^2q^2+6pq-pq^3+q^2+2q+5p^2-2p-2-2\left[\frac{2p-2q+1}{4}\right] & ,q\le p \\ (2q+4)\binom{6pq-2p}{2}-\frac{15}{2}p^5-6p^4q-\frac{3p^3}{2}-12p^2q^3+11p^2q-3p^3q-6p^2q^2+ \\ 7pq+4p^2-3p-pq^2-1 & ,q>p \end{cases}$$

2. If p is odd, then:

$$CW_{e0}(T(p,q)) = \begin{cases} (4\frac{p-1}{2}+4)\binom{6pq-2p}{2} - \frac{3}{2}pq^4 - 18p^3q^2 - 6p^2q^3 + \frac{pq^2}{2} + 8p^2q - 6p^3q - \\ 3p^2q^2 + 6pq - pq^3 + q^2 + q + 3p^2 - 2\left[\frac{2p-2q+1}{4}\right] + 8\left[\frac{p}{2}\right] + 8\left[\frac{p}{2}\right]^2 & ,q \le p \\ (2q+2)\binom{6pq-2p}{2} - \frac{15}{2}p^5 - 6p^4q - \frac{3p^3}{2} - 12p^2q^3 + 11p^2q - 3p^3q - \\ 6p^2q^2 + 7pq + 2p^2 - 3p - pq^2 + 8\left[\frac{p}{2}\right] + 8\left[\frac{p}{2}\right]^2 & ,q > p \end{cases}$$

Proof. According to the Theorem (2-7 and 1-1), we can compute the desire results easily. ■

In addition in [7], the second edge Wiener index of $TUC_4C_8(S)$ is computed.

Theorem 2-9. [7] The second edge Wiener index of $TUC_4C_8(S) = T(p,q)$ is

1. If p is even, then:

$$W_{e4}(T(p,q)) = \begin{cases} \frac{3}{2}pq^4 + 18p^3q^2 + 6p^2q^3 - \frac{pq^2}{2} - 22p^2q + 6p^3q + 21p^2q^2 - 6pq + pq^3 - \\ 2pq^2 - q^2 - 2q - p^2 + 2p + 2 + 2\left[\frac{2p-2q+1}{4}\right] & ,q \le p \\ \frac{15}{2}p^5 + 6p^4q + \frac{3p^3}{2} + 12p^2q^3 - 25p^2q + 3p^3q + 24p^2q^2 - 7pq + \\ 3p - pq^2 + 1 & ,q > p \end{cases}$$

2. If p is odd, then:

$$W_{e4}(T(p,q)) = \begin{cases} \frac{3}{2}pq^4 + 18p^3q^2 + 6p^2q^3 - \frac{pq^2}{2} - 22p^2q + 6p^3q + 21p^2q^2 - 6pq + pq^3 - \\ 2pq^2 - q^2 - q + p^2 + 2\left[\frac{2p-2q+1}{4}\right] - 8\left[\frac{p}{2}\right] - 8\left[\frac{p}{2}\right]^2 & ,q \le p \\ \frac{15}{2}p^5 + 6p^4q + \frac{3p^3}{2} + 12p^2q^3 - 25p^2q + 3p^3q + 24p^2q^2 - 7pq + 2p^2 + \\ 3p - pq^2 - 8\left[\frac{p}{2}\right] - 8\left[\frac{p}{2}\right]^2 & ,q > p \end{cases}$$

Now, The second edge reverse Wiener index, RW_{e4}, for $TUC_4C_8(S)$ is as follows.

Theorem 2-10. The second edge reverse Wiener number, RW_{e4}, of $TUC_4C_8(S)=T(p,q)$ nanotubes is

1. If p is even, then:

$$RW_{e4}(T(p,q))=\begin{cases}(2p+3)\binom{6pq-2p}{2}-\frac{3}{2}pq^4-18p^3q^2-6p^2q^3+\frac{pq^2}{2}+22p^2q-6p^3q- \\ 21p^2q^2+6pq+pq^3+2pq^2+q^2+2q+p^2-2p-2-2\left[\frac{2p-2q+1}{4}\right] & ,q\le p \\ \\ (2q+3)\binom{6pq-2p}{2}-\frac{15}{2}p^5-6p^4q-\frac{3p^3}{2}-12p^2q^3+25p^2q-3p^3q- \\ 24p^2q^2+7pq-3p+pq^2-1 & ,q>p\end{cases}$$

2. If p is odd, then:

$$RW_{e4}(T(p,q))=\begin{cases}(4\frac{p-1}{2}+3)\binom{6pq-2p}{2}-\frac{3}{2}pq^4-18p^3q^2-6p^2q^3+\frac{pq^2}{2}+22p^2q-6p^3q- \\ 21p^2q^2+6pq-pq^3+2pq^2+q^2+q-p^2-2\left[\frac{2p-2q+1}{4}\right]+8\left[\frac{p}{2}\right]+8\left[\frac{p}{2}\right]^2 & ,q\le p \\ \\ (2q+1)\binom{6pq-2p}{2}-\frac{15}{2}p^5-6p^4q-\frac{3p^3}{2}-12p^2q^3+25p^2q-3p^3q-24p^2q^2+ \\ 7pq-2p^2-3p+pq^2+8\left[\frac{p}{2}\right]+8\left[\frac{p}{2}\right]^2 & ,q>p\end{cases}$$

Proof. The desire result can be concluded according to Theorems (2-9 and 1-1) and the fact that the number of edges of $TUC_4C_8(S)$ is 6pq-2p. ■

Now, we compute the second edge complementary Wiener indices CW_{e4} of $TUC_4C_8(S)$.

Theorem 2-11. The first edge complementary Wiener indices CW_{e4} of $TUC_4C_8(S)=T(p,q)$ is

1. If p is even, then:

$$CW_{e4}(T(p,q))=\begin{cases}(2p+4)\binom{6pq-2p}{2}-\frac{3}{2}pq^4-18p^3q^2-6p^2q^3+\frac{pq^2}{2}+22p^2q-6p^3q- \\ 21p^2q^2+6pq+pq^3+2pq^2+q^2+2q+p^2-2p-2-2\left[\frac{2p-2q+1}{4}\right] & ,q\le p \\ \\ (2q+4)\binom{6pq-2p}{2}-\frac{15}{2}p^5-6p^4q-\frac{3p^3}{2}-12p^2q^3+25p^2q-3p^3q- \\ 24p^2q^2+7pq-3p+pq^2-1 & ,q>p\end{cases}$$

2. If p is odd, then:

$$CW_{e4}(T(p,q)) = \begin{cases} (4\frac{p-1}{2}+4)\binom{6pq-2p}{2} - \frac{3}{2}pq^4 - 18p^3q^2 - 6p^2q^3 + \frac{pq^2}{2} + 22p^2q - 6p^3q - \\ 21p^2q^2 + 6pq - pq^3 + 2pq^2 + q^2 + q - p^2 - 2\left[\frac{2p-2q+1}{4}\right] + 8\left[\frac{p}{2}\right] + 8\left[\frac{p}{2}\right]^2 & ,q \le p \\ (2q+2)\binom{6pq-2p}{2} - \frac{15}{2}p^5 - 6p^4q - \frac{3p^3}{2} - 12p^2q^3 + 25p^2q - 3p^3q - 24p^2q^2 + \\ 7pq - 2p^2 - 3p + pq^2 + 8\left[\frac{p}{2}\right] + 8\left[\frac{p}{2}\right]^2 & ,q > p \end{cases}$$

Proof. Due to the Theorems (2-10 and 1-1) and this fact that the number of edges of $TUC_4C_8(S)$ is equal to 6pq-2p, the results can easily obtained. ■

3 CONCLUSION

Some relations among edge reverse Wiener indices and the edge complementary Wiener indices with vertex Wiener index are concluded. And the edge versions of reverse Wiener indices are computed for trees and $TUC_4C_8(S)$ nanotubes.

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