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The Eccentric Connectivity Index of Some Special Graphs

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ABSTRACT

If G is a connected graph with vertex set V, then the eccentric connectivity index of G, $\boldsymbol{\xi^C}(G)$, is defined as $\sum_{v\in V} \deg(v)\, ecc(v)$ where $\deg(v)$ is the degree of a vertex v and $ecc(v)$ is its eccentricity. Let **A,** B and **C** are families of graphs made by joining P_n to K_m, made by putting $K_m$instead of each vertex in P_n and made by putting C_m instead of each vertex in P_n, respectively. In this paper we compute the eccentric connectivity index of these families of graphs.

Keywords: Eccentric Connectivity index, graph.

1 INTRODUCTION

A simple graph $G = (V, E)$ is a finite nonempty set $V(G)$ of objects called vertices together with a (possibly empty) set $E(G)$ of unordered pairs of distinct vertices of G called edges. In chemical graphs, the vertices correspond to the atoms and molecule, and the edges represent the chemical bonds. If $x, y \epsilon V(G)$ then the distance $d(x, y)$ between x and y is defined as the graph of a minimum path connecting x and y. The eccentric connectivity index of the molecular graph G, $\boldsymbol{\xi^C}(G)$, was proposed by Sharma, Goswami and Madan [3]. It is defined as $\sum_{v\in V(G)} \deg(v)\, ecc(v)$, where $\deg(v)$ denotes the degree of the vertex v in G and $ecc(v) = Max\{d(x, v)|x\epsilon V(G)\}$, see [1,2] for more details.

We denote the complete graph, the cycle and the path of order n by K_n, C_n and P_n, respectively. In the Section 2, the eccentric connectivity index of some special graphs is computed.

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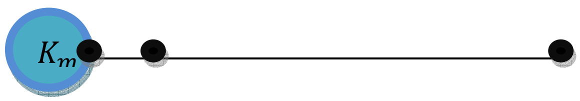
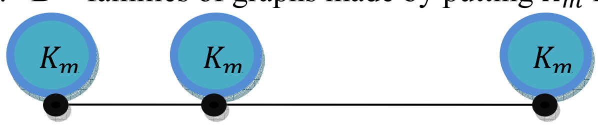
2 THE ECCENTRIC CONNECTIVITY INDEX OF SOME SPECIAL GRAPHS

In this section we compute the eccentric connectivity index of the special graphs A**,** B and C as follows:

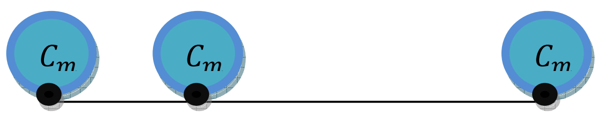
a. **A** = families of graphs made by joining K_m to P_n;



b. **B** = families of graphs made by putting K_m instead of each vertex in P_n;



c. **C** = families of graphs made by putting C_m instead of each vertex in P_n.



Note. For a path of order n we have;

$$\xi^C(P_n) = \begin{cases} \frac{1}{2}(3n^2 - 6n + 4), if\ n\ is\ even \\ \frac{3}{2}(n-1)^2, \quad\quad if\ n\ is\ odd \end{cases}$$

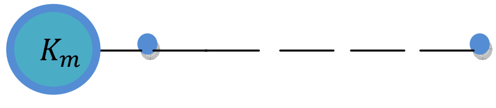
Definition 1. Let G be a graph and W be a subset of $V(G)$. We define

$$\xi^C(W:G) := \sum_{w \in W} deg_G(w) ecc_G(w)$$

Definition 2. Let m, n be two positive integers. Define $\alpha_{m,n}$ as

$$\alpha_{m,n} = (m-1)^2 n + m(n-1)$$

Let G be a graph made by joining the first vertex of P_n to K_m. Then we have $\alpha_{m,n} = \xi^C(V(K_m):G)$



P_n

If m is even, then we define $\beta_{m,n}$ as

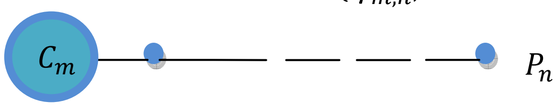
$$\beta_{m,n} = 2\left(\frac{m}{2} + n - 1\right) + 3\max\left(\frac{m}{2}, n-1\right) + 4\sum_{j=2}^{\frac{m}{2}} \max\left(\frac{m}{2}, j+n-2\right).$$

If m is odd, then we define $\gamma_{m,n}$ as

$$\gamma_{m,n} = 3\max\left(\frac{m-1}{2}, n-1\right) + 4\sum_{j=2}^{\frac{m+1}{2}} \max\left(\frac{m-1}{2}, j+n-2\right).$$

Let G be a graph made by joining the first vertex of P_n to C_m. Then we have

$$\xi^C(V(C_m):G) = \begin{cases} \beta_{m,n}, & m \text{ is even} \\ \gamma_{m,n}, & m \text{ is odd} \end{cases}$$



Theorem 1. Let A, B and C be the mentioned graphs. Then the eccentric connectivity indices of these graphs are as follows;

1. $\xi^C(\mathbf{A}) = \xi^C(P_{n+1}) + m(mn-n-1) - 2(n-1)$.

2. $$\xi^C(\mathrm{B}) = \begin{cases} 2\left(\alpha_{m,n+1} + \sum_{i=\frac{n+4}{2}}^{n}(\alpha_{m,i})\right) + \frac{3}{4}(n^2-2n), & \text{if } n \text{ is even} \\ \alpha_{m,\frac{n+3}{2}} + 2\left(\alpha_{m,n+1} + \sum_{i=\frac{n+5}{2}}^{n}(\alpha_{m,i})\right) + \frac{3n^2-6n-1}{4}, & \text{if } n \text{ is odd} \end{cases}.$$

3. $$\xi^C(\mathbf{C}) = \begin{cases} 2\gamma_{m,n+\frac{m-1}{2}} + \sum_{i=2}^{\frac{n+1}{2}}\left(\gamma_{m,n-i+\frac{m+1}{2}}\right) + \sum_{i=\frac{n+3}{2}}^{n-1}\left(\gamma_{m,i+\frac{m-1}{2}}\right) + \frac{3n^2-12n+2mn-4m+11}{4}, & m \text{ is odd}, n \text{ is odd} \\ 2\gamma_{m,n+\frac{m-1}{2}} + \sum_{i=2}^{\frac{n}{2}}\left(\gamma_{m,n-i+\frac{m+1}{2}}\right) + \sum_{i=\frac{n+2}{2}}^{n-1}\left(\gamma_{m,i+\frac{m-1}{2}}\right) + \frac{3(n-2)^2+2m(n-2)}{4}, & m \text{ is odd}, n \text{ is even} \\ 2\beta_{m,n+\frac{m}{2}} + \sum_{i=2}^{\frac{n}{2}}(\beta_{m,n-i+\frac{m+2}{2}}) + \sum_{i=\frac{n+2}{2}}^{n-1}(\beta_{m,i+\frac{m}{2}}) + \frac{3n^2+2mn-10n-4m+8}{4}, & m \text{ is even}, n \text{ is even} \\ 2\beta_{m,n+\frac{m}{2}} + \sum_{i=2}^{\frac{n+1}{2}}(\beta_{m,n-i+\frac{m+2}{2}}) + \sum_{i=\frac{n+3}{2}}^{n-1}(\beta_{m,i+\frac{m}{2}}) + \frac{3n^2-10n+2mn-4m+7}{4}, & m \text{ is even}, n \text{ is odd} \end{cases}.$$

Proof.

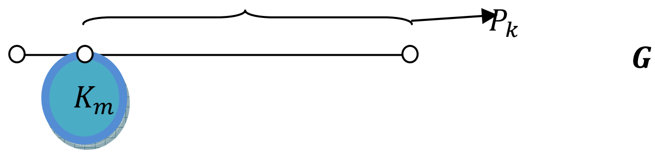
1. According to the definition of $\alpha_{m,n}$, in order to compute the eccentric connectivity index of **A**, it is enough to compute this index for the vertices left on the path of $n+1$ vertices. But in this way two vertices of K_m are computed also in computing the indices of the vertices of the path, so we must minus them. In this case we have:

$$\xi^C(\mathbf{A}) = \xi^C(P_{n+1}) + \alpha_{m,n} - 2(n-1) = \xi^C(P_{n+1}) + m(mn-n-1) - 2(n-1).$$

2. First suppose that n is even. Because of the symmetric condition, it is enough to compute the eccentric connectivity index for half of this graph and then makes it two times. We have two steps to compute:

i. $\alpha_{m,n+1;}$

ii. $\xi^C(K_m: G)$, for each $\frac{n+4}{2} \leq k \leq n$. This equals to $\alpha_{m,k} + k - 1$, for each k.



Now by computing these two steps we have;

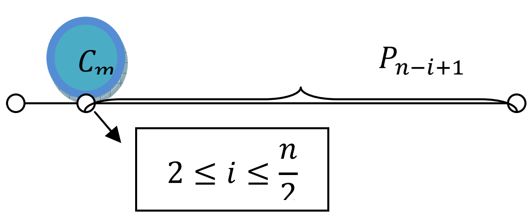
$$\xi^C(\mathrm{B}) = 2\left(\alpha_{m,n+1} + \sum_{i=\frac{n+4}{2}}^{n}(\alpha_{m,i} + i - 1)\right) = 2\left(\alpha_{m,n+1} + \sum_{i=\frac{n+4}{2}}^{n}(\alpha_{m,i})\right) + \frac{3}{4}(n^2 - 2n).$$

Now suppose that n is odd, so we have a central vertex. So we must do the same as we did for the case n is even, and compute this index for the central vertex, also. So we have

$$\xi^C(\mathrm{B}) = \alpha_{m,\frac{n+3}{2}} + \frac{n+1}{2} + 2\left(\alpha_{m,n+1} + \sum_{i=\frac{n+5}{2}}^{n}(\alpha_{m,i} + i - 1)\right)$$
$$= \alpha_{m,\frac{n+3}{2}} + 2\left(\alpha_{m,n+1} + \sum_{i=\frac{n+5}{2}}^{n}(\alpha_{m,i})\right) + \frac{3n^2-6n-1}{4}.$$

3. First let m be even. We have three steps:

i. $\xi^C(C_m: G)$, in which G is the following graph. This equals to $\beta_{m,n-i+1+\frac{m}{2}} + n - i + \frac{m}{2}$.



ii. Computing the previous index when $\frac{n}{2} < i < n$, which equals to $\beta_{m,i+\frac{m}{2}} + i + \frac{m}{2} - 1$;

iii. And finally for $i = 1, n$, we get $\beta_{m,n+\frac{m}{2}}$.

So,

$$2\beta_{m,n+\frac{m}{2}}+\sum_{i=2}^{\left[\frac{n+1}{2}\right]}\left(\beta_{m,n-i+1+\frac{m}{2}}+n-i+\frac{m}{2}\right)+\sum_{i=\left[\frac{n+1}{2}\right]+1}^{n-1}\left(\beta_{m,i+\frac{m}{2}}+i+\frac{m-2}{2}\right)=$$

$$\begin{cases}2\beta_{m,n+\frac{m}{2}}+\sum_{i=2}^{\frac{n}{2}}\left(\beta_{m,n-i+\frac{m+2}{2}}\right)+\sum_{i=\frac{n+2}{2}}^{n-1}\left(\beta_{m,i+\frac{m}{2}}\right)+\frac{3n^2+2mn-10n-4m+8}{4}, & m\ is\ even, n\ is\ even\\ 2\beta_{m,n+\frac{m}{2}}+\sum_{i=2}^{\frac{n+1}{2}}\left(\beta_{m,n-i+\frac{m+2}{2}}\right)+\sum_{i=\frac{n+3}{2}}^{n-1}\left(\beta_{m,i+\frac{m}{2}}\right)+\frac{3n^2-10n+2mn-4m+7}{4}, & m\ is\ even, n\ is\ odd\end{cases}.$$

The case m is odd is the same, just we need to compute the index for the central vertex also.

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